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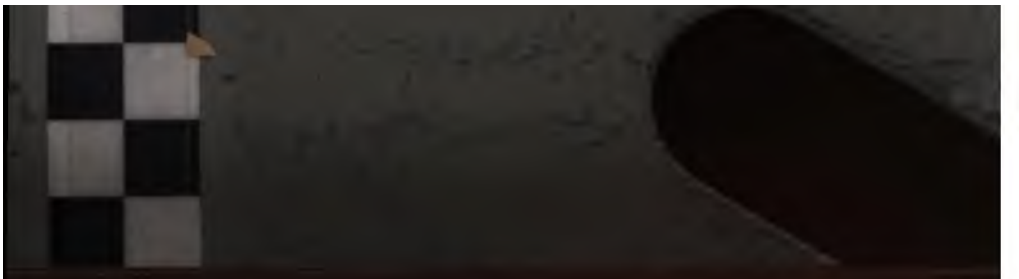

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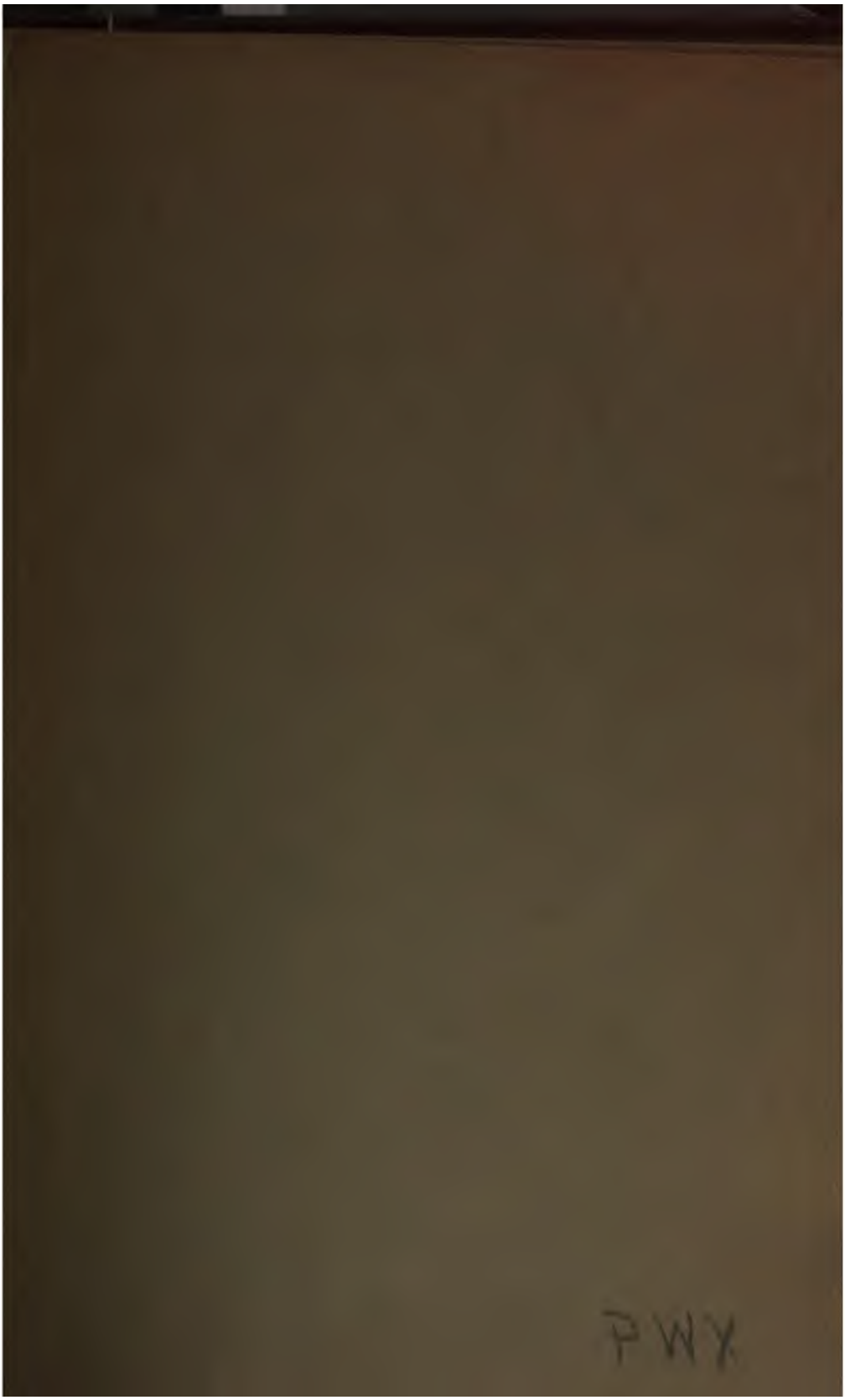
PWY

THE

CHARACTERS OF CRYSTALS

AN INTRODUCTION

TO

PHYSICAL CRYSTALLOGRAPHY

BY

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PREFACE.

I have attempted, in this book, to describe, simply and concisely, the methods and apparatus used in studying the physical characters of crystals, and to record and explain the observed phenomena without complex mathematical discussions. In the last chapter the graduate course in Physical Crystallography given in Columbia University has been outlined.

The works most consulted have been *Grundriss der Physikalischen Krystallographie*, 1896, by T. Liebisch; *Physikalische Krystallographie*, 1895, by P. Groth; *Traité de Crystallographie*, 1884, by E. Mallard; *Treatise on Crystallography*, 1839, by W. H. Miller; *Crystallography*, 1895, by N. Story-Maskelyne. Footnote references are also given to many important articles.

It is hoped the book will be found useful to organic chemists, geologists, mineralogists and others interested in the study of crystals.

A. J. M.

MINERALOGICAL DEPARTMENT,
COLUMBIA UNIVERSITY, NEW YORK CITY,
MARCH, 1899.

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ERRATA.

In Figs. 41, 89, 190 the axis of composite symmetry should be dotted.
Fig. 180 is upside down.
P. 50 for $\{h\ h\ 2\ h\ i\}$ read $\{h\ h\ 2\ h\ i\}$,
P. 53, line 7, for dehexagonal read dihexagonal.
P. 56, line 4, for Fig. 111 read Fig. 162.
P. 82, line 20, for OA = 1.732 read OA × 1.732.
P. 84, line 5, for O read O′.
P. 96, last line, for Weinschenk read Weinshenk.
P. 114 line next to last for $\mu\mu = .009$ read $\Delta = 530\ \mu\mu$.
P. 123, line 28, for two circular read two equal circular.
P. 142, line 5 from the bottom, for acute read obtuse.
P. 145, lines 3 and 4, for 146 and 147 read 118 and 119.
P. 146, lines 14 and 22, for 146 read 118.

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PART I. GEOMETRICAL CHARACTERS.

CHAPTER I.

INTRODUCTORY.

It s a general property of definite chemical substances to assume at solidification regular forms bounded by planes and observation has proved that the forms which occur are related to each other and characteristic of the substance.

The geometric study of crystals has for its purpose the grouping of crystals into series, each of which shall consist of the forms in which one substance can appear; the determination of the angles between the faces of crystals, and from these the elements and sym-

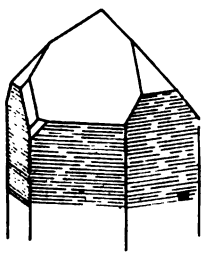


FIG. 1.

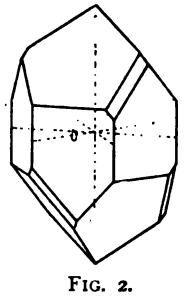


FIG. 2.

bols of the faces, or conversely the determination of the form and angles from the elements and the face symbols.

The term crystal originally meant the angular forms of the substance called rock crystal, Figs. 1 and 2, which, according to PLINY,* was only "water frozen by the most excessive cold and found only in places where snow is changed into ice." The an-

* Quoted by Romé Delisle, *Essai de Crystallographie* (1772), p. 3.

gular shapes of this substance and of garnet,* beryl and possibly diamond, were known to the ancients, but were regarded as accidents and no general property was suspected.

In 1568 WENTZEL JAMITZER,† a Nuremburg goldsmith, developed in perspective over one hundred and forty simple and complex shapes from the geometric tetrahedron, octahedron, cube, dodecahedron and icosahedron by replacing all similar edges and angles by one or more planes, and the famous astronomer KEPLER, in 1619, developed a similar series of figures. Many of these shapes correspond to crystals, and the method of modification may have suggested to Romé Delisle the method used by him more than a century later.

The development of chemistry from alchemy was accompanied by the study of many salts, the solutions of which, on evaporation, yielded regular and constant shapes which were suspected to be to

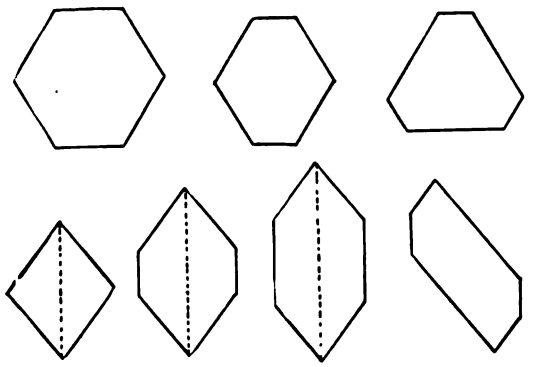


FIG. 3.

some extent at least characteristic of the salt, for LIBAVIUS,‡ in 1597, stated that the nature of the saline components of a mineral water could be ascertained by an examination of the crystalline form of the salts left on the evaporation of the water.

In 1669 NICOLAS STENO,§ a Danish anatomist, announced that if

* "There is also an incombustible stone found about Miletum which is of an angular shape, and sometimes regularly hexangular; they call this also a carbuncle." Theophrastus's *History of Stones*. Trans. by Sir John Hill, p. 77.

†Perspectiva Corporum Regularum, Quoted in Marx's *Geschichte der Krystalkunde*.

‡Roscoe & Schorlemmer's *Treatise on Chemistry*, I. 705.

§De solido intra solidum naturaliter contento dissertationis prodromus. Florentiæ, 1669: English translation, London, 1671.

different specimens of rock crystal were examined, it would be found that in spite of the variation in the relative size of the faces and in the shape of the crystal there was *no variation in the angles between the faces.* This he illustrated by figures of sections at right-angles to a prism edge and others of sections at right angles to an edge between a prismatic and a pyramidal face, as shown in Fig. 3.

A statement showing a very considerable advance from this was made by DOMINICO GULIELMINI in 1704, who asserted that every salt had its peculiar shape which never changed and that even in imperfect and broken crystals the angles were constant.

TORBERN BERGMAN* records that his pupil GAHN having broken a piece of Dogtooth Spar observed that it divided into little rhombohedra. In studying this, Bergman found that by laying together these rhombohedra in certain ways he could build up either a hexagonal prism terminated by this form or the common scalenohedron of calcite, or a form like the rhombic dodecahedron, all of which are shown in Fig. 4.

FIG. 4.

From this he deduced that there was a relation between outer form and inner structure and that the constituent parts of all crystals could be referred to a very small number of primitive forms which could be found by breaking the crystals.

In the preparation of models of crystal forms ROMÉ DELISLE, about 1783, measured† the interfacial angles directly with an application goniometer devised by Carangeot for this purpose. He described over four hundred crystal forms and formulated the now universally accepted law of constancy of interfacial angles as follows:‡ "*In spite of the numberless variations of which the primitive form of a salt or a crystal is capable, one thing never varies, but is always constant in each species, namely: The angle of incidence or the respective inclination of the faces to each other.*"

The law of symmetry may also be said to be based on a law of Delisle. "Every face has a similar face parallel to it."

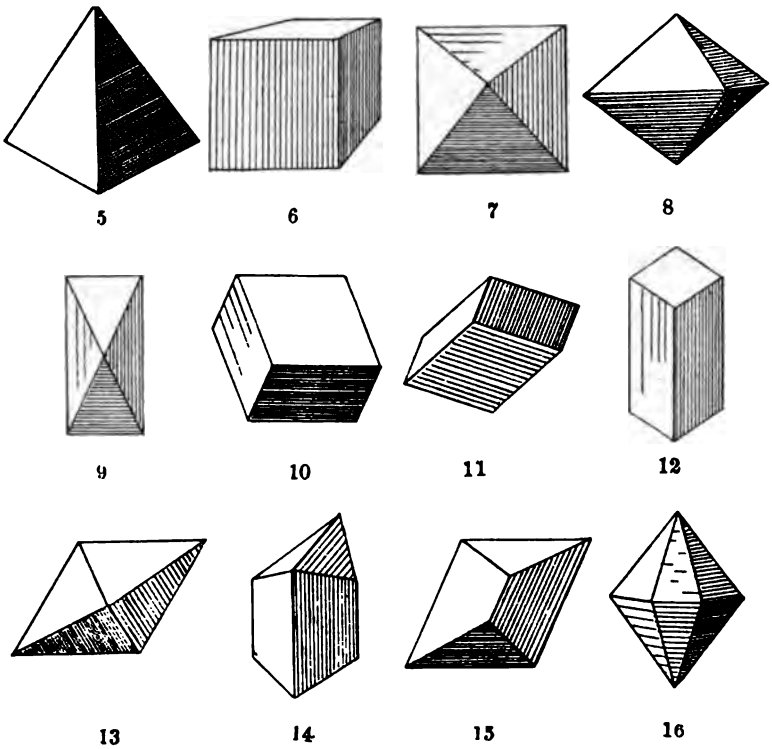
---

* *De Formis Crystallorum* cited in von Kobell's *Geschichte der Mineralogie.*—p. 81.

† The few measurements of crystals which had thus far been made were measurements of plane angles only, which varied with differences in development so that the interfacial angles calculated from them were often incorrect.

‡ Cristallographie, ou description der formes propres à tous les corps du règne minéral. Paris, 1783. p. 70.

Delisle also developed Bergman's idea of primitive form and by a method similar to that of Jamitzer derived a series of secondary forms from each primitive by replacing the edges or angles of the primitive form by single planes or groups of planes in such a way that equivalent geometric parts were similarly treated. He referred all forms to six classes of primitive form: *The regular tetrahedon*, Fig. 5, and *the cube*, Fig. 6, which permit of no varieties, and *the rectangular octahedron*, Figs. 7, 8, 9; *the rhomboidal parallelopipedon* Figs. 10, 11, 12; *the rhomboidal octahedron* Figs. 13, 14, 15, and *the dodecahedron with triangular faces*, Fig. 16, all of which were susceptible of many varietions according to their angles.



As an example of the method of derivation there could be derived from the cube, Fig. 17, Fig. 18 by truncating each edge, Fig. 19 by bevelling each edge, Fig. 20 by truncating each solid angle, Fig. 21 by replacing each solid angle by three planes, each cutting equal lengths from two edges and a different length from the third, Fig. 22 by replacing each solid angle by six planes, each

cutting three unequal distances on the three edges, and many others. Similarly all the secondary forms of any compound could be derived from a primitive form, and thus connecting them in a definite series. Any series could, however, be derived from almost any member of the series in the manner described and the primitive forms could only be chosen arbitrarily.

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The establishment of crystallography upon a mathematical basis is chiefly due to the Abbé Haüy, who, apparently in ignorance of the discoveries of Bergman, had his attention directed to the internal structure of crystals by the accidental dropping of a six-sided prism of calcite which broke into rhombohedral fragments. He found that this property of "cleavage" was a general one and that* the crystals were built up of molecules of the shape of the cleavage forms. Although in many instances no cleavage was found or cleavage only in one direction, which yielded no solid, he assumed that such solids did exist in all crystals and were the true primitive forms upon which the other secondary forms depended and that the shape in such cases could be determined by striations and other markings on the faces or by analogy between the shapes of the secondary forms and similar forms of other crystals which did show cleavage.

Like Delisle he made six groups of primitive forms, of which only two were identical with those of Delisle namely the tetrahedron, Fig. 5, and the dodecahedron with triangular faces, Fig. 16.

The forms, 1. Paralleopipedon. Including *cube; right prisms*

* Essai d'une theorie sur la structure des cristaux 1784. *Journal de Physique* XLIII, p. 103.

Also in Gren's *Neues Journal der Physik*, 1795, VII., p. 418.

with bases square, rectangular, rhombic, oblique; *oblique prisms* with bases rhombic, Fig. 28; rectangular, oblique; *rhombohedron* obtuse; acute. 2. REGULAR TETRAHEDRON. 3. OCTAHEDRON WITH TRIANGULAR FACES. Including the regular Octahedron and the octahedra with bases rectangular, square, rhombic. 4. SIX-SIDED PRISM. 5. DODECAHEDRON WITH RHOMBIC FACES. 6. DODECAHEDRON WITH TRIANGULAR FACES.

The existence of cleavages other than those producing the primitive form led Haüy to assume the existence of still simpler shapes which he called integrant molecules. These he limited to three kinds. The tetrahedron, the trigonal prism and the parallelopipedon.

With crystals of any substance Haüy discovered that all forms other than the primitive form could be exactly imitated by building on the faces of the primitive form successive plates or layers of integrant molecules each successive layer regularly diminishing by the abstraction of one or more rows, either parallel to each edge, or to the diagonals of the faces of the primitive form, or parallel to some intermediate line. A rhombic dodecahedron might, for instance, be found to cleave with equal ease in three directions at right angles to each other. The integrant molecules would then, according to Haüy, be cubes and the kernel or prim-

FIG. 23.

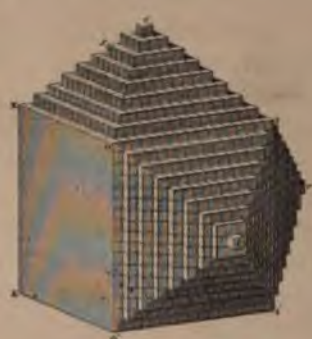

FIG. 24.

itive form also a cube, Fig. 23; then, as perfect cleavage requires that the minute integrant molecules be so piled as not to break joints, the structure would be similar to that shown in Fig. 24, in which the successive layers piled on top of any face of the cubic primitive form regularly diminish one row at a time on each edge.

For only with such a rate of decretion can the pyramidal planes sOI and tOI unite to one plane sOIt and the diedral angle between alternate planes be ninety degrees. As the number of plates added would be very large and the little cubic molecules too small to be separately visible the steps would appear to lie in the planes.

Another example may be given to show that this method of building yields forms corresponding to actual crystals. Fig. 25 shows a form observed on cobaltite, in which the diedral angle at the edge, $p\,q$, is 126° 52′. Assuming a cubic kernel, Fig. 26. Fig. 27 shows the structure according to Haüy, in which the decretions

FIG. 25.

FIG. 26.

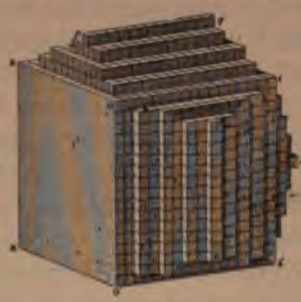

FIG. 27.

correspond to a triangle with a base of 2 and a perpendicular of 1, that is, the angle at the base has a tangent equal to 0.5, or is 26° 34′. The angle at $p\,q$ is twice the complement of this; that is, 126° 52′.

The decretion* was symmetrical that is it was repeated on all similar parts of the kernel and Haüy's experiments showed that the secondary planes usually resulted from the subtraction of one or of two rows, and were always according to some simple rational number never to his knowledge exceeding four. This is essentially the basis of the law of rational indices, the fundamental law of crystallography.

PROF. BERNHARDI, of Erfurt, pointed out† that the primitive form should be chosen, not as with Haüy, from molecules of nature, but according to convenience and fitness, and that

*The law of symmetry was stated by Haüy as follows: "It consists in this, that any one method of decretion is repeated on all those parts of the nucleus of which the resemblance is such, that one can be substituted for the other by changing the position of this nucleus with respect to the eye, without it (the nucleus) ceasing to be presented in the same aspect." *Memoire sur une loi de Cristallisation*, 1815.

†Von Kobell's *Geschichte der Mineralogie*, p. 197.

the paralellopipedon was not a satisfactory form for calculation in many cases, as it was often an open form the height of which could only be determined from some secondary face, and he recommended the choice of closed primitive forms only and suggested seven, six of which are still regarded as the simplest forms in the crystal systems, namely: *Cube, Rhombohedron, Square Octahedron, Rhombic Octahedron, Rhomboidal Octahedron, Triple Rhomboidal Octahedron*, the seventh form was a *rectangular octahedron* which can be referred to the *rhombic octahedron.*

PROF. WEISS, of Berlin, devised a purely geometric mode of treatment in which he discarded entirely the idea of primitive form, saying* "that Haüy's hypotheses of decreted rows entangled the problem with self-created difficulties" and that the "mechanical-atomic presentation which Haüy deduced should be stripped away so that the acquired knowledge of the mathematical relation should be more clearly seen." Weiss assumed the existence of certain fundamental lines or axes passing through a common center and made four groups: 1st, with three equal axes at right angles to each other; 2d, with three axes at right angles to each other, of which two were equal; 3d, with three axes at right angles, and all different lengths; 4th, with three equal axes at sixty degrees to each other in one plane and one at right angles to these, but of different length.

He deduced all the primitive forms of Haüy by constructing planes which passed: First—Through ends of three lines. Second—Through ends of two of the lines and parallel to a third. Third—Through an end of one of the lines and parallel to two of them. By taking points along each of these lines at twice, three times and four times, etc., the original length, and constructing planes in the same way as before, he obtained all the secondary forms.

The lengths of the semi-axes of the primitive forms were called a, b and c, and the position of any face was denoted by the ratio of the intercepts in terms of a, b and c; thus, $2a : b : 3c$ or $a : 3b : 2c$, and so on. This method is still the simplest and most satisfactory if only an elementary knowledge of type forms is desired, but is cumbersome and tedious in calculation.

HAUSMANN applied spherical trigonometry to crystallographic calculations in 1803 and BERNHARDI† in 1808 pointed out that it

* Uebersichtliche Darstellung der verschiedene Abtheilung der Krystallisations-Systeme. *Denkschrift der Berliner Akad. der Wissen.* 1814-15. p. 298.

† *Gehlen's Journal*, 1808, 2, 378.

would be better to determine trigonometrically the relations between the lines from a common point, normal to each face, as these were the directions of attraction and growth.

PROF. MOHS, of Freiberg, grouped* all forms in four SYSTEMS, the *Cubic*, *Pyramidal*, *Rhombohedral* and *Prismatic*, each consisting of a series of forms geometrically derived from a "fundamental form" by modifying planes which cut distances from certain lines in the ratio of whole numbers. The fundamental forms were the simplest closed forms and were not selected from cleavage or structure, but purely for geometric simplicity. In 1822 he proved the existence of two more Systems, the *Monoclinic* and *Triclinic*, the forms of which had previously been considered as partial forms of the other four systems.

F. C. NEUMANN in 1823 proposed a graphic method in which the crystal faces were indicated by the points in which radii drawn normal to the faces met the surface of a circumscribing sphere.

In 1825 WHEWELL† showed that if a solid angle of the primitive form was taken as the origin, and the edges as cordinates, with lengths x, y, z, then any secondary face would be expressed by the equation $\frac{x}{h}.\frac{y}{k}.\frac{z}{l} = m$, in which m was not dependent upon the inclination of the face, and, therefore, the quantities $\left\{\frac{1}{h}.\frac{1}{k}.\frac{1}{l}\right\}$ or say $\{p, q, r\}$ could represent the face. These are essentially the reciprocals of the Weiss intercepts, and are the indices used by Miller.

By far the most important advance since Haüy was the method developed‡ by PROFESSOR W. H. MILLER, of Cambridge University, in which, by spherical trigonometry, a series of simple symmetrical expressions were deduced for the normal angles in terms of the reciprocal intercepts of Whewell, and the positions of faces were indicated by stereographic projection of the points in which radii perpendicular to the faces meet the surface of the sphere. The method is still the best, and is gradually supplanting all others except for elementary work.

* Treatise on Mineralogy, or the Natural History of the Mineral Kingdom. (Haidinger's translation), Edinburgh, 1825.

† A general method of calculating the angles made by any planes of crystals. Rev. W. Whewell, Transactions Royal Society, 1825, CXV., 87–180.

‡ A treatise on Crystallography, 1839.

CHAPTER II.

THE GENERAL GEOMETRIC PROPERTIES OF CRYSTALS.

A PLANE OF SYMMETRY is a plane through the centre which divides the crystal so that either half is the mirrored reflection of the other, and every line perpendicular to the plane (and within the solid) connects corresponding points of the solid and is bisected by the plane. Every plane of symmetry is necessarily parallel to a possible crystal face, for if two faces are symmetrical to it, it must pass through their edge, that is to be in the same zone. So, also, with a second pair of faces. But any plane lying in two known zones has rational indices (p. 11) and is a possible crystal face, as will be shown later.

One plane of symmetry may occur alone. If two planes occur they must be at right angles, and they make necessary a third at right angles to both.

AN AXIS OF SYMMETRY is a line through the centre such that a revolution of 180° or less around it will bring a face into coincidence with the original position of an equivalent face.

The intersection of two planes of symmetry is necessarily an axis of symmetry. Every axis of symmetry is either parallel to an edge between two possible faces or normal to a possible face. It has been proved* that that there can be only four kinds of axes of symmetry: *Binary*, *Ternary*, *Quaternary* and *Senary* in which equivalent faces become coincident by revolutions of 180°, 120°, 90° and 60° respectively. These may occur alone or in combinations or with planes of symmetry.

There may be distinguished three methods by which equivalent planes equally distant from the centre may be made to coincide:

1st. By reflection in a plane of symmetry.

2d. By rotation around an axis of symmetry.

* Groth's *Physikalische Krystallographie*, p. 313, III. edition.

3d. By composite symmetry or simultaneous rotation around an axis and reflection in a plane.

Composite symmetry can occur with a binary axis, and plane perpendicular thereto; it is impossible with a ternary axis, and with quarternary or senary axes is equivalent to simple symmetry with binary or ternary axes respectively.

Axes. The faces of crystals are defined in position by referring them to imaginary lines called axes.* The selection is largely arbitrary, but with the object of securing the simplest indices. For this purpose they must be lines parallel to the edges between possible planes and lines as important to the symmetry as possible. Generally three directions are selected which are parallel to the intersections of three occurring or possible faces.

Fig. 28.

Let $X\bar{X}$ $Y\bar{Y}$ $Z\bar{Z}$, Fig. 28, be three such axes and let all distances be measured on these lines from their origin or common intersection O, distances in the direction OX, OY or OZ being regarded as positive and those in the directions $O\bar{X}$, $O\bar{Y}$ or $O\bar{Z}$ as negative.

Any plane as A B C would be determined in space by its intercepts OA, OB, OC, on these axes, or in angular position by the ratios of these intercepts, and as in crystals there is constancy in inclinations of planes to each other and therefore to the axes, but no constancy in the linear dimensions, some expression for the relative instead of the absolute values of the intercepts is used as a symbol, which may be the intercepts expressed as a proportion or some condensed expression derived from the ratios of the intercepts.

The Fundamental Law or Law of Rational Indices.

Experience teaches that a simple relation exists between the intercepts of different planes of the crystals of any chemical substance, and that *if the axial intercepts of any face be divided by the corresponding intercepts of any other face the quotients will be only simple rational numbers.*

* Equivalent axes are necessarily surrounded by the same number of faces placed in the same way.

For instance, if the intercepts of some face A B C are

$$OA : OB : OC = a : b : c,$$

then these values divided by the intercepts of any other face H K L will yield quotients

$$\frac{a}{OH} : \frac{b}{OK} : \frac{c}{OL} = h : k : l,$$

in which h, k and l are simple rational numbers such as $\frac{1}{2}$, $\frac{1}{3}$, 1, 2, 3.

PARAMETERS.

Some selected plane A B C is called the parametral face or unit face, and the values $a\ b\ c$, which express the simplest ratios of its intercepts, are called the parameters of the crystal.

INDICES.

It is always possible to express the ratios between h, k and l by three whole numbers because, as just stated they are simple rational numbers, and if fractional they may always be cleared of fractions without changing the ratios, for evidently

$$h : k : l = mh : mk : ml$$

The simplest* whole numbers which express the ratios of h, k and l are called the indices of the face, and are always written in the same order, the first referring to the intercept on the axis $X\bar{X}$ the second to that on $Y\bar{Y}$, the third to that on $Z\bar{Z}$. A bar over an index, as $\bar{k}$, indicates that the intercept is negative. As indicating a face they should be written (hkl), though frequently the parenthesis is omitted; but if used as a symbol of a form they should be written $\{hkl\}$.

Indices and intercepts are inversely proportional, for from the equation above we obtain:

$$OH = \frac{a}{h}, OK = \frac{b}{k}, OL = \frac{c}{l},$$

in which a, b and c are constant for all faces of a crystal.

If a face is parallel to an axis its intercept on that axis is infinite and its index is zero; for example, if $\frac{a}{h} = \infty$, then $h = 0$ and the symbol is $(0kl)$.

* Experience shows that the faces which most frequently occur will, with proper selection of the parametral plane, have as indices 0 or 1 or rarely 2.

The values of the intercepts OH, OK, OL, become the parametral values only when $hkl = 111$. For with these values only can we obtain:

$$OH = \frac{a}{h} = a, \quad OK = \frac{b}{k} = b, \quad OL = \frac{c}{l} = c.$$

Determination of the Elements of a Crystal.

The three axial planes and the "parametral face" constitute the *elementary planes*. These are chosen to yield the simplest indices and are usually planes of cleavage, or twinning or gliding planes, and are preferably planes of symmetry.

In the most general case there are five undetermined elements, namely:

The angles between the axes, YZ or α, XZ or β, XY or γ.

The ratio between the parameters a, b, c, in which $b = 1$.

1° *Determination of the angles between the axes.*

These may be determined most simply from the measured angles between the axial planes, as follows:

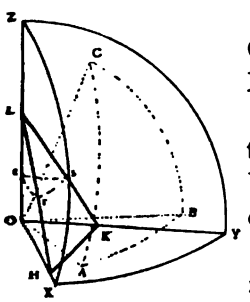

Fig. 29.

Let the surface of a sphere, described around O, the centre of the crystal, meet the axes in X, Y and Z, Fig. 29.

Construct A B C the polar triangle of X Y Z, then will A be the pole of the axial plane YOZ, B the pole of XOZ, and C the pole of XOY.

The sides of A B C therefore measure the normal angles between the axial planes, that is between the planes (010), (100), (001) and these angles are determined by measurement

$$AB = (100)-(010), \quad BC = (010)-(001), \quad AC = (100)-(001).$$

The angles of the triangle can therefore be calculated by formula

for instance: $$\cos CAB = \frac{\cos BC - \cos AB \cos AC}{\sin AB \sin AC}$$

Since A B C and X Y Z are polar triangles

$$\alpha = YZ = 180° - CAB, \quad \beta = XZ = 180° - CBA,$$
$$\gamma = XY = 180° - ACB.$$

The angles between the axes are therefore determined.

EXAMPLE. In Axinite by measurement it is found that

		log. cos.	log. sin.
A B or (100 — 010) = 48° 21′ 8″	whence	9.82253	9.873465
B C or (010 — 001) = 97° 50′ 8″	"	9.13459	9.99593
A C or (100 — 001) = 93° 48′ 56″	"	8.82311	9.99904

$$l \cos CAB = 9.09181$$

$$CAB = 82° 54′ 13″ \text{ or } 97° 5′ 47″$$

$$\alpha = 97° 5′ 47″ \text{ or } 82° 54′ 13″$$

Similarily β and γ can be determined

2° *Determination of the parameter ratios.*

In Fig. 29 let HKL be any plane cutting the three axes and with known indices *hkl*.

The spherical triangle *rst* described from L as a centre can be solved because its angles are

t=(100 —(010) s= (100) — (*hkl*) and r= (010) — (*hkl*), all of which may be measured.

The sides, *tr* and *ts*, may be calculated by formula, for instance:

$$\cos tr = \frac{\cos s + \cos t \cos r}{\sin t \sin r}.$$

These sides measure angles in the plane triangles H O L and K O L, respectively, and in each of these one other angle has already been determined. The triangles may, therefore, be solved for the *relative* lengths of their sides—that is, for the intercepts of the plane.

For instance, in K O L, the angle at L is measured by *ts*, and the angle at O by Y Z or α, hence the angle at K is 180°—(α+*ts*), and the sides are given by the formula,

$$O\,L : O\,K = \sin (180° - \alpha - ts) : \sin ts$$

Similarly,

$$O\,L : O\,H = \sin (180° - \beta - tr) : \sin tr$$

EXAMPLE:

Given t=100° 41′, s=59° 10′, r=76° 33′, α=82°21′, β=73° 11′. Required a, b and c.

$$\cos tr = \frac{\cos s + \cos t \cos r}{\sin t \sin r} = \frac{.5125 + .1854 \times .2335}{.98215 \times .9726} = .5819,$$

$$tr = 54^\circ\ 35'.$$

$$\cos ts = \frac{\cos r + \cos t \cos s}{\sin t \sin s} = \frac{.2335 + .1854 \times .5125}{.98215 \times .8587} = .3895$$

$$ts = 67^\circ\ 5'.$$

In triangle K O L O L : O K = sin (180° — 82° 21′ — 67° 5′): sin 67° 5′, = sin 30° 34′ : sin 67° 5′ = 0.5085 : .9205 = .5525 : 1.

In triangle H O L O L : O H = sin (180° — 73° 11′ — 54° 35′): sin 54° 35′ = sin 52° 14′ : sin 54° 35′ = 0.7905 : .8150 = .5525 : .5696

Hence O H : O K : O L = .5696 : 1 : .5525 = $a : b : c$.

CHAPTER III.

SPHERICAL PROJECTION.

Imagine a sphere described around the centre of a crystal with any radius and radii drawn normal to each face.

The point in which the radius normal to any crystal face meets the surface of the sphere, is called the *pole* of the face, and is denoted by the symbol of the face.

Planes which intersect in parallel edges will evidently have their normals in one plane and their poles in the circle which it cuts from the sphere of projection. Such a series of planes constitute a *Zone*, the plane of the normals is the *Zone Plane*, the circle is the *Zone Circle*, and the line through the centre parallel to the face and edges of the zone is the *Zone Axis*.

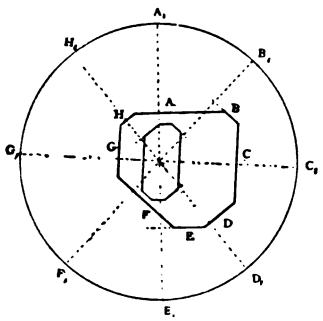

FIG. 30.

Fig. 30 represents the section made by a zone plane, the normals to the faces A, B, C, etc., meet the zone circle in the poles A_1, B_1, C_1, etc.

The same radii are evidently normal to the crystal faces of the enclosed ideal form in which equivalent faces are equally distant from the centre. Hence the poles on the surface of the sphere, in their arrangement will reveal the hidden regularity of unequally developed crystals.

Fig. 30 shows, also, that the arc of the zone circle between any two poles measures the normal angles, the supplements of the angles between the corresponding faces. Different intersecting zone circles give spherical triangles, the sides of which are normal angles, which can be solved by simple formulæ, provided certain parts have been determined by measurement or previous calculation.

A number of simple relations between the faces in zones have been deduced by means of which it is possible to greatly reduce the number of necessary direct measurements and to simplify the calculations.

Symbol for a Zone Axis.

The direction of intersection* of two crystal faces (hkl) and $(h'k'l')$ is $[\mathbf{uvw}]$ in which $\mathbf{u} = kl' - lk'$, $\mathbf{v} = lh' - hl'$, $\mathbf{w} = hk' - kh'$.

These values may be obtained by cross multiplication of the twice written indices, striking off end terms and reading down alternately from left to right and from right to left, thus:

$$\begin{array}{c|cccccc|c} h & k & & l & & h & k & l \\ & & \times & & \times & & \times & \\ h' & k' & & l' & & h' & k' & l' \end{array}$$

As all the terms are whole numbers the values of **u**, **v** and **w** will be also. A zone is designated by this symbol **[uvw]** or by the symbols of two of its planes $[hkl, pqr]$, or by letters designating the faces [P, Q], always enclosing with the parallel bars.

Equation for Zone control or Condition that a face may belong to a zone.

If a face (pqr) lies in a zone **[uvw]** its indices must satisfy† the equation $p\mathbf{u} + q\mathbf{v} + \mathbf{w}r = 0$.

If two indices of a face are known, and the zone is known, the third index may therefore sometimes be found.

Example. By test with a reflection goniometer the face $(3k1)$ is found to be in the zone $[111]$; substituting in the above equation $3\times1 + k\times1 + 1\times1 = 0$ whence $3 - k - 1 = 0$ and $k = 2$.

Face in two zones.

The indices of a face‡ which lies in the zones **[uvw]** and **[u'v'w']** will be $h = \mathbf{uv'} - \mathbf{vu'}$, $k = \mathbf{vw'} - \mathbf{wv'}$, $l = \mathbf{wu'} - \mathbf{uw'}$.

These values may be obtained by cross multiplication.

Example.—By test a face (hkl) is found to lie in the zones $[2\bar{1}0]$ and $[01\bar{2}]$, required the values of h, k and l.

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* Miller's *Treatise on Crystallography*, p. 7.

† Ibid p. 10.

‡ Ibid, p. 8.

$$\begin{array}{c|cccc|c} 2 & \bar{1} & 0 & 2 & \bar{1} & 0 \\ & \times & \times & \times & & \\ 0 & 1 & \bar{2} & 0 & 1 & \bar{2} \end{array} \quad h = 2-0,\ k = 0+4,\ l = 2-0,$$

that is, $(h\,k\,l) = (242) = (121)$.

To Find a Fourth Face in a Zone.*

Let $A = (efg)$, $B = (h\,k\,l)$ and $C = (p\,q\,r)$ be known faces of a zone, the poles of which lie in the order named.

To find the position of any fourth face, $D = (m\,n\,o)$, if its indices are known.

$$\frac{A\,C}{C\,D}(\cot A\,C - \cot A\,D) = \frac{A\,B}{B\,D}(\cot A\,B - \cot A\,D).$$

The values of $\frac{A\,C}{C\,D}$ and $\frac{A\,B}{B\,D}$ are such as result by cross multiplication of pairs of corresponding indices:

$$\frac{A\,C}{C\,D} = \begin{array}{c} e\ \ f \\ \times \\ p\ \ q \\ p\ \ q \\ \times \\ m\ \ n \end{array} = \frac{eq-fp}{pn-qm};\ \text{or}\ \begin{array}{c} f\ \ g \\ \times \\ q\ \ r \\ q\ \ r \\ \times \\ n\ \ o \end{array} = \frac{fr-gq}{qo-rn};$$

$$\text{or}\ \begin{array}{c} e\ \ g \\ \times \\ p\ \ r \\ p\ \ r \\ \times \\ m\ \ o \end{array} = \frac{er-gp}{po-rm}$$

$$\frac{A\,B}{B\,D} = \begin{array}{c} e\ \ f \\ \times \\ h\ \ k \\ h\ \ k \\ \times \\ m\ \ n \end{array} = \frac{ek-fh}{hn-km};\ \text{or}\ \begin{array}{c} f\ \ g \\ \times \\ k\ \ l \\ k\ \ l \\ \times \\ n\ \ o \end{array} = \frac{fl-gk}{ko-ln};$$

$$\text{or}\ \begin{array}{c} e\ \ g \\ \times \\ h\ \ l \\ h\ \ l \\ \times \\ m\ \ o \end{array} = \frac{el-gh}{ho-lm}$$

Very frequently two of the three identical ratios are indeterminate.

* Groth's *Physikalische Krystallographie*, III. ed., p. 584.

EXAMPLE.

In a crystal of pyroxene, given: A = (*efg*) = (100), B = (*hkl*) = (101), C = (*pqr*) = (001), D = (*mno*) = (301)., A B = 49° 39′, A C = 73° 59′. Required A D.

By trial the first and third ratios are found to be indeterminate, but from the third we obtain

$$\frac{AC}{CD} = \frac{1.1 - 0.0}{0.1 - 1.\bar{3}} = \frac{1}{3}; \quad \frac{AB}{BD} = \frac{1.1 - 0.1}{1.1 - 1.\bar{3}} = \frac{1}{4}$$

substituting in the equation

$$\frac{1}{3}(\cot 73° 59' - \cot AD) = \frac{1}{4}(\cot 49° 39' - \cot AD)$$

$$1.1504 - 4 \cot AD = 2.5487 - 3 \cot AD. \quad \cot AD = 1.3983$$

$$AD = 144° 26'$$

ZONE OF TWO PINACOIDS.

Every face in the zone will have that index zero which is zero in both pinacoids. For example if a face lies in the zone of [100, 010] its third index must be zero for the zone symbol is [001] p. 17 and $h.0 + k.0 + l.1 = 0$ can be true only if $l = 0$.

ZONE THROUGH ONE PINACOID.

The ratio of the two indices which are zero for the pinacoid, is constant for all faces of the zone. For example the symbol of the zone [100, hkl] is [$0l\bar{k}$]. Any third face (pqr) must satisfy the equation $p0 + q\,l - r\,k = 0$, that is $ql = rk$ or $\frac{k}{l} = \frac{q}{r}$

ZONES IN WHICH TWO INDICES ARE CONSTANT.

If two faces have two corresponding indices in each with the same ratio, all faces in their zone will have those two indices in that ratio. For example, the symbol of the zone of the faces (123) and (245) is [$\bar{2}10$] A face (hkl) to be in this zone must satisfy the equation $-2h + k + 0\,l = 0$, that is $2h = k$.

CHANGING AXES.*

If three edges are preferred to those originally selected as axes directions proceed as follows:

From the original indices of the *new* axial planes determine the symbols of their intersections (that is, their zone symbols), [uvw], [$u_1v_1w_1$] [$u_2v_2w_2$].

* Miller *Treatise on Crystallography*, p. 17.

Then, if the indices of any face referred to the old axes is (hkl), its new indices h_1, k_1 and l_1 will have the following values:

$$h_1 = h\mathbf{u} + h\mathbf{v} + h\mathbf{w}$$
$$k_1 = k\mathbf{u}_1 + k\mathbf{v}_1 + k\mathbf{w}_1$$
$$l_1 = l\mathbf{u}_2 + l\mathbf{v}_2 + l\mathbf{w}_2.$$

CHANGING PARAMETERS.

hkl the indices of a face referred to parameters a, b, c,
$h_1k_1l_1$ " " " the " " " " " a_1, b_1, c_1,

$\frac{a_1}{h_1} = \frac{a}{h}$, $\frac{b_1}{k_1} = \frac{b}{k}$, $\frac{c_1}{l_1} = \frac{c}{l}$ then if this face is chosen as the parametral plane $a_1 = \frac{a}{h}$, $b_1 = \frac{b}{k}$, $c_1 = \frac{c}{l}$ and any other face, the indices of which were pqr, will receive new indices $p_1q_1r_1$, in which

$$p_1 = \frac{pa_1}{a}, \quad q_1 = \frac{qb_1}{b}, \quad r_1 = \frac{rc_1}{c}.$$

STEREOGRAPHIC PROJECTION.

The sphere containing the poles of the crystal faces is most conveniently represented in Stereographic Projection.

Some important plane through the centre of the sphere is selected as a plane of projection, and all poles of the sphere are projected in this plane and fall within the limits of the so-called " PRIMITIVE CIRCLE."

For instance, let Fig. 31 represent the upper half of a crystal of cassiterite within a sphere of projection. The crystal faces a, d, g, k, m., etc., will have their poles in the points A, D, G, k_1, m_1, etc., where the normal radii intersect the spherical surface.

Let the equatorial plane be the plane of projection, let the south pole be the point of sight, then if lines are drawn from the south pole to the various poles in the northern hemisphere, they will all pierce the equatorial plane at points A, D, G, K, M, etc., in the primitive circle which are the projections of the poles.

PRINCIPAL CHARACTERISTICS OF STEREOGRAPHIC PROJECTION.

1° All zone circles at right angles to the primitive circle are

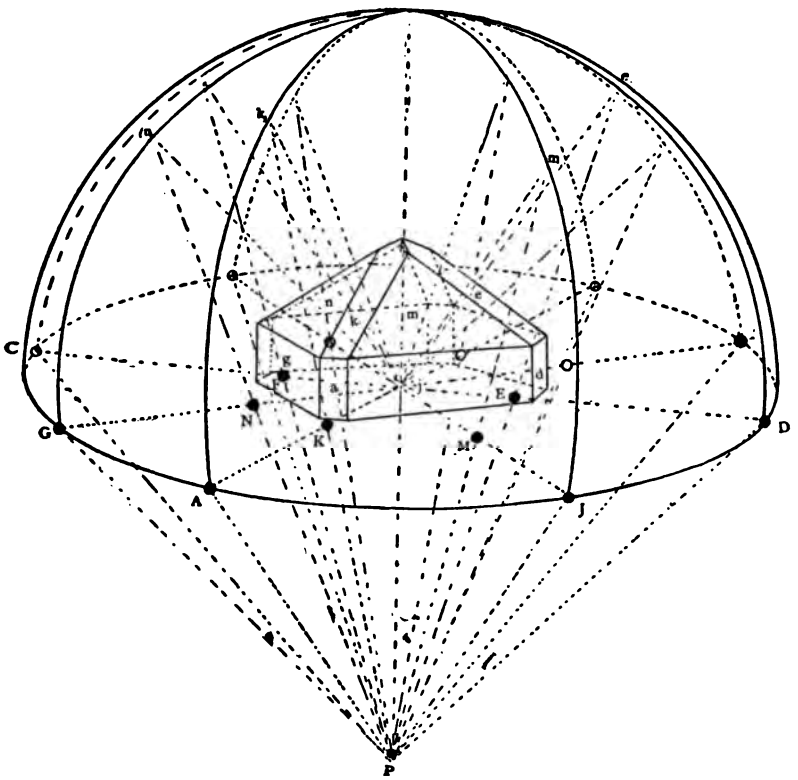


FIG. 31.

projected as diameters, all others, as arcs of circles,* cutting the primitive circle in the extremities of a diameter.

2° *If the pole* (*F. Fig. 33*) *of a zone circle* CPC_1 *is united with the poles of two faces* (*P and Q*) *of the zone by straight lines, and these prolonged to the circumference, they cut from it an arc equal to the normal angle between the two faces.*†

Free-hand construction in many instances serves to give a comprehensive view of the symmetry and relations of the faces and zones. For accurate construction a few rules are needed, which are based upon characteristics 1 and 2.

PROBLEM 1.

Given the projection of a zone circle, to find that of its pole.‡

* For demonstration, Story-Maskelyne's Crystallography, p. 30.

† Ibid, p. 33.

‡ Miller's *Treatise on Crystallography*, p. 133.

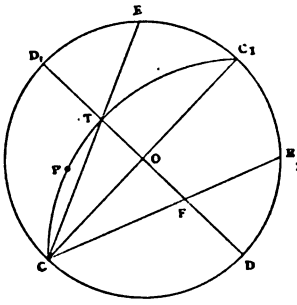


FIG. 32.

In Fig. 32. Given C P C_1 to find F.

C P C_1 intersects the primitive circle in the diameter C C_1. Its pole must lie on that circle which is projected as the diameter D D_1 normal to C C_1 and at 90° from the point T where the two circles intersect. Draw C E through T, lay off E E_1 = 90°, draw E_1C, then is F the required pole because it is a point on D D_1 and by the second characteristic it lies at a quadrants distance from T.

PROBLEM 2.

Given the projection of the pole of a great circle to draw the circle.

In Fig. 32 given F to find C P C_1.

Draw a diameter D D_1 through F and another C C_1 normal to this. Through F draw C E_1, make E E_1 90°, draw E C, then are T, C and C_1, three points of the zone desired. Through these pass the arc of a circle.

PROBLEM 3.

In any zone given the angle between two faces and the projection of one of them to find that of the other.

In Fig. 33, given the zone CPC_1, the face P, and the angle P to Q=*a*. To find Q,

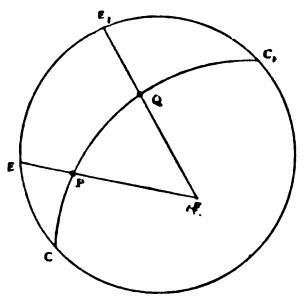


FIG. 33.

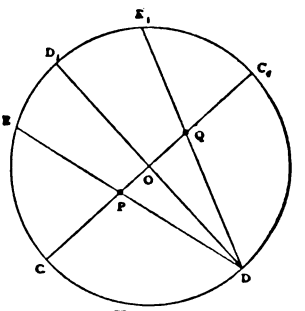


FIG. 34.

Find F the pole of the zone C P C_1, as in problem 1. Draw F E through P, make E E_1 equal *a*, and draw E_1 F. Then by the second characteristic, Q is the desired projection.

In Fig 34, is represented the special case in which the given

zone C P C_1 is normal to the primitive circle. In this case the pole of the zone falls at D, the extremity of the diameter normal to C C_1.

If $a = 180°$, the second face will lie outside the primitive circle. The construction is not changed.

PROBLEM 4.

*Given the projections of two faces in a zone to find the projection of the zone circle.**

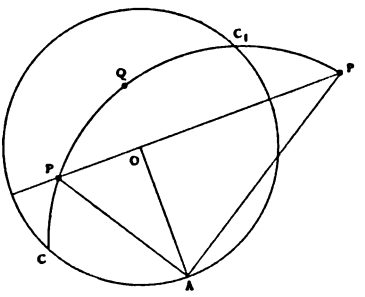


FIG. 35.

In Fig. 35, given P and Q to find C P C_1.

Draw a diameter through P. The face P_1 opposite P, will be on it.

Draw O A perpendicular to this diameter, draw P A, and from A, a line perpendicular to P A. The intersection of this perpendicular with the diameter through P, will be P_1; the projection of the pole of a face opposite P, that is 180° from P, in the same zone.

The arc of a circle through the three points, P, Q and P_1, will be the desired circle.

EXAMPLES. Application of preceding constructions to the projection (Fig. 36) of the cassiterite crystal (Fig. 31).

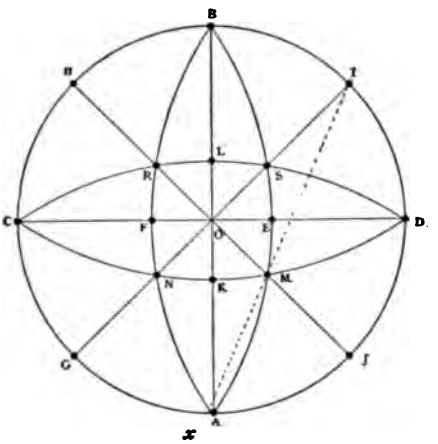


FIG. 36.

The poles lying in the primitive circle are separated by arcs

*Miller's *Treatise on Crystallography*, p. 133.

equal to the true normal angles. The poles of all other faces i this instance lie in zone circles normal to the primitive circl therefore projected as diameters. The true position of any one c these is found by Problem 3. For example: M — J or (110) - (111) = 46° 27′. Lay off J x = 46° 27′, draw x I to the pole

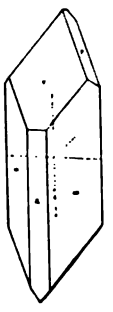


FIG. 37.

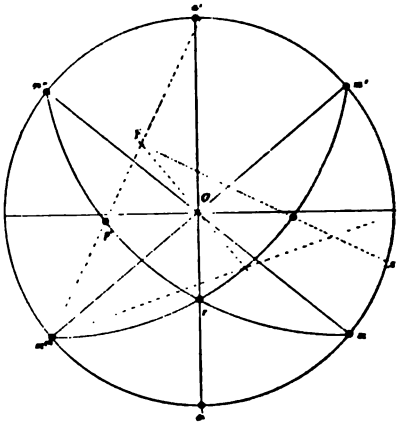


FIG. 38.

of the circle J H cutting the latter at M, the required projectio The pole R is in the zone of M and J, and being an equivalent fac O R = O M.

Draw zone circles through C M D, B M A, C R D and B R A

The poles of N and S will lie at the intersections as show For the second order forms K lies in the zone [A B] and in the zor [M N], hence is at K. Similarly the faces e, l and f have poles L and F.

A crystal of barium chlorate shown in Fig. 37 yields the pr jection Fig. 38. The plane of projection is taken normal to th zone [m, a].

Given $m - a$ = 48° 53½′, $m - r$ = 41° 51½′, $m - p$ = 56° 3 In Fig. 37 lay off arcs equal 48° 53½′ each side of a and a' d termining m, m', m'', m'''. Lay off an angle equal 41° 51½′ an draw a line (not shown) to pole of the zone [a a'], intersecting th latter at r. By trial p lies in the zone [m''' r], construct this zor circle, find its pole, F by Problem. Lay off m' s = 56° 39′, dra s F, then, by Problem 3, p is the desired pole; p' is in the zone [m symmetrical to p.

CHAPTER IV.

THE THIRTY-TWO CLASSES OF CRYSTALS.

In this classification, following Professor Groth, the conception of hemihedral and tetartohedral forms is abandoned because the geometrically connected whole and partial forms *have no structural connection and are incapable of occurrence upon crystals of the same substance.*

Each occurring form is, therefore, considered to be complete and independent, and with a grade of symmetry which is not in every case determinable from a consideration of the grouping of the faces, but involves the far wider conception that two directions are not structurally equivalent *unless they are equivalent with respect to all properties*, and that two forms geometrically identical are not structurally so if in corresponding directions there is revealed an essential difference in behavior with polarized light or etching or pyroelectricity or any other test, the results of which depend upon the manner the crystal molecules are built together.

With this conception the same geometric form may occur on crystals structurally different, and is to be regarded as in each case a limit form of geometrically distinct general forms. If a class be made of each conceivable variation of general form and its geometric limit forms, two great essentials will be fulfilled:

1. All known and some unknown forms will be classed.

2. Each class will consist of forms capable of occurring upon crystals of the same substance, and will contain all the forms which can occur on these crystals.

There have been distinguished thirty-two grades or classes of symmetry which may be united into six systems by grouping together classes in which the selected axes of reference are geometrically similar.

In the stereographic projection of the general form given under each class all full lines, whether diameters, circles or arcs of circles represent planes of symmetry and the small black ellipses, triangles, squares and hexagons represent the axes of binary, ternary quaternary and senary symmetry.

TRICLINIC SYSTEM.

This system must include all crystallographic forms which can only be referred to three non-equivalent axes, Fig. 39, at oblique angles, α, β, and γ, to each other.

The selection of axes is arbitrary, two edges are chosen as directions of the basal axes, a and b. Two planes from the zones of these edges are chosen as {100} and {010} and their intersections are the vertical axes c.

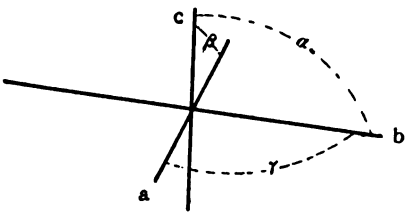


FIG. 39.

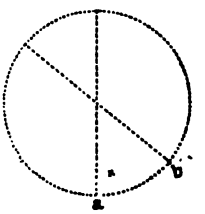


FIG. 40.

1. UNSYMMETRICAL CLASS.

Without either planes or axes of symmetry. The projection, Fig. 40, shows that the symmetry of the class is satisfied with one face for the most general form. EXAMPLE.—Calcium thiosulphate, $CaS_2O_3.6H_2O$.

2. CLASS OF PINACOIDS.

With composite symmetry to a binary axis and a plane normal thereto.

As shown in projection, Fig. 41, any upper face indicated by X would by simultaneous rotation around the axis and reflection in the plane coincide with the diametrically opposite lower face, that is, the symmetry of the class is satisfied by two faces for the most

general form or PINACOID (Tetarto Pyramid), Fig. 42. EXAMPLES.—Albite, cyanite and chalcanthite.

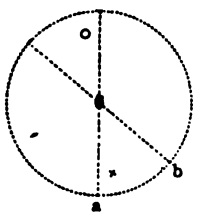


FIG. 41.

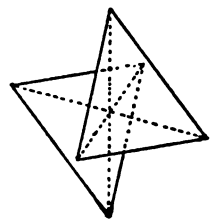


FIG. 42.

THE SIX LIMIT FORMS.

In each class there are six limit forms corresponding to six special positions of the faces of the general form. In Class 2 these will be pairs of parallel planes, and in Class 1 single planes only.

Position of any Face.	*Symbol.*	*Class 1.*	*Class 2.*
1. Parallel to *a* and *b*.	{001}	PLANE of Fig. 43.	PINACOID Fig. 43.
2. Parallel to *a* and *c*.	{010}	PLANE of Fig. 44.	PINACOID Fig. 44.
3. Parallel to *b* and *c*.	{100}	PLANE of Fig. 45.	PINACOID Fig. 45.
4. Parallel to *a*.	{0*kl*}	PLANE of Fig. 46.	PINACOID Fig. 46.
5. Parallel to *b*.	{*h*0*l*}	PLANE of Fig. 47.	PINACOID Fig. 47.
6. Parallel to *c*.	{*hk*0}	PLANE of Fig. 48.	PINACOID Fig. 48.

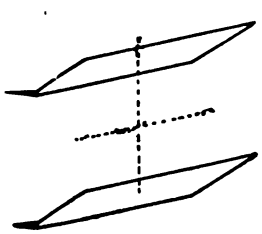


FIG. 43.

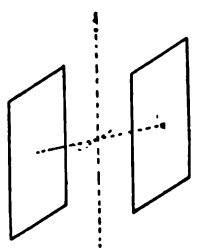


FIG. 44.

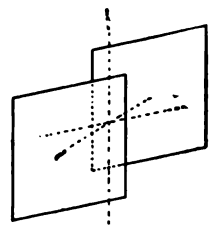


FIG. 45.

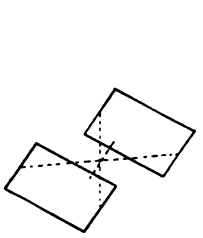


FIG. 46.

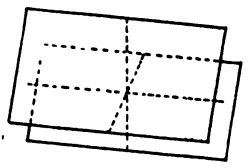


FIG. 47.

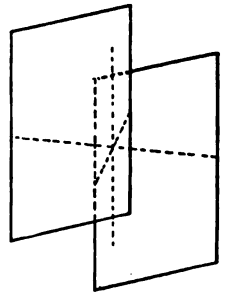


FIG. 48.

PROJECTION AND CALCULATION OF TRICLINIC FORMS.

THE PROJECTION is conveniently made upon a plane normal to the vertical axis c. All planes in the zone of c as (100), (010); (110), being projected in the primitive circle at the measured angles apart.

The pole of any face P will lie at the intersection of two circles found as follows:* Their centers are on prolongations of diameters through A(100) and B(010) and distant from the center O of the primitive circle respectively $OK = \frac{r}{PA}$, $OL = \frac{r}{PB}$; and their radii are respectively $Kp = r \tan PA$, $Lp = r \tan PB$.

Most of the poles are found by zones and problem 3, p. 22.

THE CALCULATIONS are principally solutions of spherical triangles† and of equations which connect measured angles with

*Groth, *Physikalische Krystallographie*, p. 580, III. ed.

† *For right-angled spherical triangles*

$C = 90°$ a, b, c = sides opposite angles A, B, C.

$$\sin A = \frac{\sin a}{\sin c} = \frac{\cos B}{\cos b}, \qquad \cos A = \frac{\tan b}{\tan c} = \cos a \sin B,$$

$$\tan A = \frac{\tan a}{\tan b}, \qquad \cos c = \cos a \cos b = \cot A \cot B$$

In oblique angled spherical triangles

A, B, C denote angles, a, b, c, opposite sides, $\frac{A+B+C}{2} = S$, and $\frac{a+b+c}{2} = s$

$$\frac{\sin A}{\sin a} = \frac{\sin B}{\sin b} = \frac{\sin C}{\sin c}$$

$$\cos A = -\cos B \cos C + \sin B \sin C \cos a$$

$$\cos a = \cos b \cos c + \sin b \sin c \cos A$$

$$\sin \frac{A}{2} = \sqrt{\frac{\sin (s-b) \sin (s-c)}{\sin b \sin c}} \qquad \sin \frac{a}{2} = \sqrt{\frac{-\cos S \cos (S-A)}{\sin B \sin C}},$$

$$\cos \frac{A}{2} = \sqrt{\frac{\sin s \sin (s-a)}{\sin b \sin c}} \qquad \cos \frac{a}{2} = \sqrt{\frac{\cos (S-B) \cos (S-C)}{\sin B \sin C}}$$

indices and elements. Wherever possible the zonal equations, pp. 16–20, are used to simplify the calculations.

DETERMINATION OF ELEMENTS.

The general determination of pp. 13–15 requiring the measurement of five angles between four planes, no three of which lie in the same zone, is followed as in the example there given.

GENERAL EQUATION BETWEEN AXES AND INDICES.

Let any plane the indices of which are *hkl* meet the axes in *HKL*, let the axes meet the surface of a sphere described around *O*, in *XYZ* and let the normal *Op* to *HKL* meet the sphere in *P*. Fig. 49.

From a section through *OHp* it is evident that $Op=OH \cos PX$, and, similarly, $Op=OK \cos PY$ and $Op=OL \cos PZ$. Equating, $OH \cos PX=OK \cos PY=OL \cos PZ$.

From page 13, we have

$$OH=\frac{a}{h},\quad OK=\frac{b}{k},\quad OL=\frac{c}{l}$$

Substituting,

$$\frac{a}{h}\cos PX=\frac{b}{k}\cos PY=\frac{c}{l}\cos PZ$$

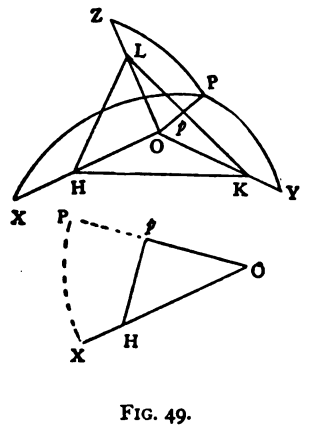


FIG. 49.

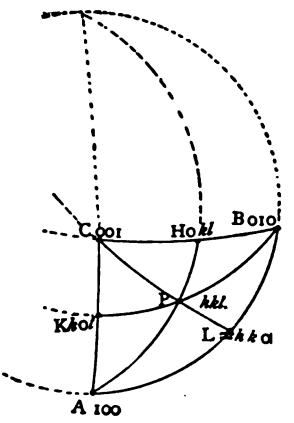


FIG. 50.

TO DETERMINE POSITION OF ANY POLE *P*.

Let Fig. 50 show the poles in projection. From the following equations the position of *P* will result if the indices and elements

are known, or, conversely, the indices will result* if the position is known in one zonal circle.

$$\frac{\sin AB \sin CH}{\sin CA \sin BH} = \frac{\sin CAP}{\sin BAP} = \frac{ck}{bl}$$

$$\frac{\sin BC \sin AK}{\sin AB \sin CK} = \frac{\sin ABP}{\sin CBP} = \frac{al}{ch}$$

$$\frac{\sin CA \sin BL}{\sin BC \sin AL} = \frac{\sin BCP}{\sin ACP} = \frac{bh}{ak}$$

If any five of the arcs in the following are known, the sixth results

$$\sin BH \sin CK \sin AL = \sin CH \sin AK \sin BL.$$

IF THE ELEMENTS ARE UNKNOWN the indices of a face lying in a known zone may be determined by measuring the angle to one face in that zone, and as three faces must already be known, substituting in the formula to find fourth face in a zone, p. 18. Conversely, if the indices of such a plane are known, its position may be calculated.

TO FIND† THE ARC JOINING ANY TWO POLES P AND P'.

Calculate by preceding equations the distances of P and P' from one of the poles A, B or C, and the angles that these distances make with one of the adjacent sides of ABC, therefore, the angle that they make with each other. From two sides and included angle calculate the third side PP'.

MONOCLINIC SYSTEM.

All forms in this system must be referable to three non-equivalent axes, Fig. 51, two oblique to each other, the third, normal to their plane.

Conventionally the normal or ortho axis b extends from right to left, either of the other axes is made the vertical, c, and the third the clino, a, dips downward from back to front. The acute angle between the vertical and clino axes is β.

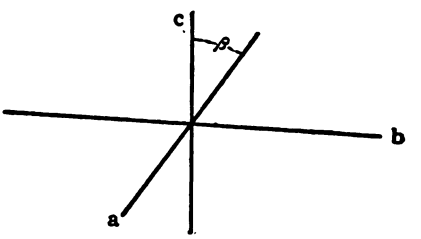


FIG. 51.

* Story-Maskelyne's *Crystallography*, p. 430.

† Miller's *Crystallography*, p. 98.

The system comprises three classes, in which are possible many series each including all the forms which can be referred to the same value for β and to the same parameters a, b, c.

3. Class of the Monoclinic Sphenoid.

With one axis of binary symmetry. As shown in the projection upon (010) Fig. 52, the pole of any face by rotation 180° around the binary axis must reach the pole of an equivalent face. The two faces satisfy the symmetry for the most general form or Sphenoid, Fig. 53. Examples.—Tartaric acid, milk sugar.

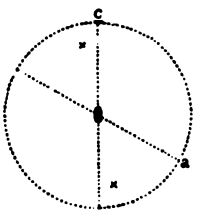


Fig. 52.

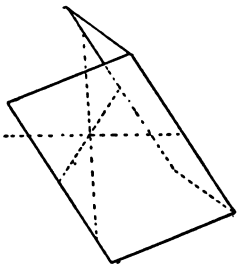


Fig. 53.

4. Class of the Monoclinic Dome.

With one plane of symmetry. As shown in the projection of the general form, Fig. 54 any face reflected in the plane of symmetry coincides with an equivalent opposite face. The two faces satisfy the symmetry of the class for the most general form or Dome, Fig. 55. Example.—Potassic tetrathionate $K_2S_4O_6$.

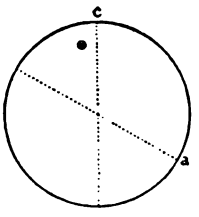


Fig. 54.

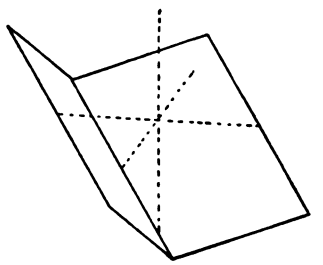


Fig. 55.

5. Prismatic Class.

With one plane of symmetry, Fig. 56, at right angles to an axis of binary symmetry. As shown in projection, Fig. 57, any face

the pole of which is marked × by rotation of 180° around the binary axis coincides with an equivalent face in the alternate octant, these reflected in the plane of symmetry coincide with faces the poles of which are marked by a circle.

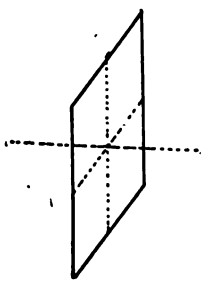


FIG. 56.

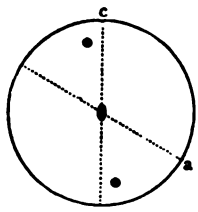


FIG. 57.

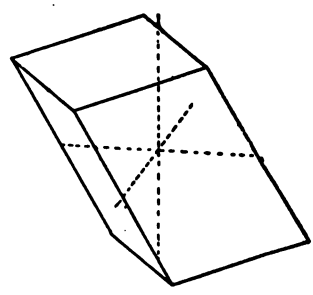


FIG. 58.

That is four planes satisfy the symmetry for the most general form or PRISM, Fig. 58. EXAMPLES.—Pyroxene, orthoclase, gypsum.

THE SIX LIMIT FORMS.

In each class there are six limit forms corresponding to special positions of the faces of the general form. These may be tabulated as follows:

Position of Any Face and its Pole.	Symbol.	Name of Form.	Classes to which Form Belongs.
1. Parallel to *a* and *b*. . . . . Poles projected* on the primitive circle	{001}	PINACOID, Fig. 59. (Basal Pinacoid.)	3, —, 5.
		BASAL PLANE. One face of Fig. 59.	—, 4, —.
2. Parallel to *a* and *c* . . . . Poles projected at centre . .	{010}	PINACOID, Fig. 60. (Clino Pinacoid.)	—, 4, 5.
		PLANE, One face of Fig. 60.	3, —, —.
3. Parallel to *b* and *c* . . . . Poles at intersections of primitive circle and horizontal diameter	{100}	PINACOID, Fig. 61. (Ortho Pinacoid.)	3, —, 5.
		PLANE, One face of Fig. 61.	—, 4, —.
4. Parallel to *a* Poles are on diameter from (001) . . .	{0*kl*}	PRISM, Fig. 62. (Clino Dome.)	—, —, 5.
		DOME, Fig. 65.	—, 4, —.
		SPHENOID, Fig. 67.	3, —, —.
5. Parallel to *b* Poles are on primitive circle.	{*hol*}	PINACOID, Fig. 63. (Hemi Ortho Dome.)	3, —, 5.
		PLANE, One face of Fig. 63.	—, 4, —.
6. Parallel to *c* Poles are on Horizontal diameter . . .	{*hko*}	PRISM, Fig. 64. (Monoclinic Prism.)	—, —, 5.
		DOME, Fig. 66.	—, 4, —.
		SPHENOID, Fig. 68.	3, —, —.

* Plane of projection the pinacoid (010).

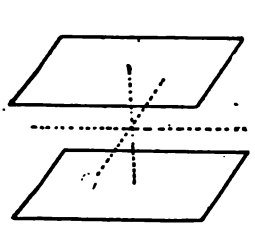


FIG. 59.

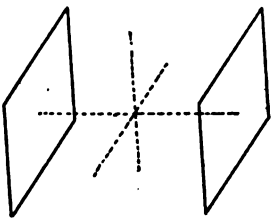


FIG. 60.

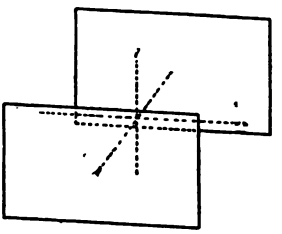


FIG. 61.

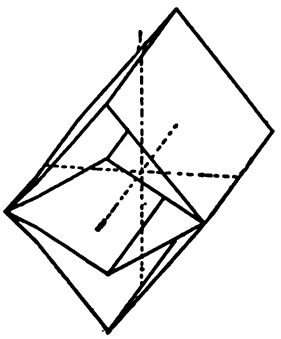


FIG. 62.

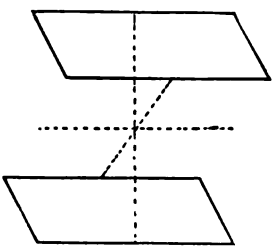


FIG. 63.

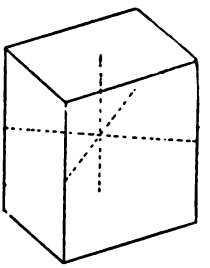


FIG. 64.

LIMIT FORMS OF CLASS 5.

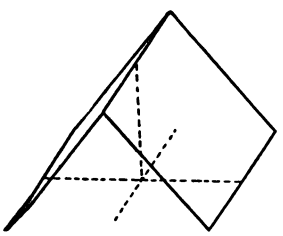


FIG. 65.

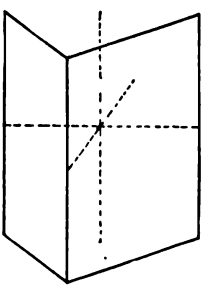


FIG. 66.

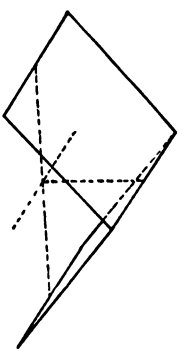


FIG. 67.

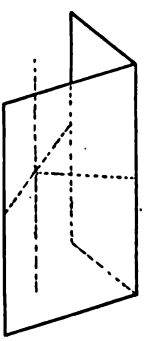


FIG. 68.

OTHER LIMIT FORMS WHICH ARE NEW SHAPES.

PROJECTION AND CALCULATION OF MONOCLINIC FORMS.

The plane of projection may be the pinacoid (010), which is a plane of symmetry in classes 4 and 5, when the poles of all

planes in the zone of b will lie in the primitive circle as given in preceding table. Or the projection may be made, as in the triclinic system, upon a plane normal to the vertical axis, in which case the poles of planes in the zone of b will be projected on the vertical diameter as described in example, p. 24.

The poles will be found as in the triclinic system.

When a plane of symmetry appears as a diameter in the projection, all poles and indices will be symmetrical to this diameter.

DETERMINATION OF ELEMENTS.

In Fig. 29, p. 13, B and Y will coincide and BC, AB and t become 90°. Then $AC=\beta$. In the spherical triangle rst,

$$\cos tr = \frac{\cos s}{\sin r};\ \cos ts = \frac{\cos r}{\sin s};\ \frac{OL}{OK} = \frac{\cos ts}{\sin ts} = \cot ts$$

and $\dfrac{OL}{OH} = \dfrac{\sin(180°-AC-tr)}{\sin tr}$. If P is the parametral plane $OH:OK:OL = a:b:c$, if not, $a = OH.h$, $b = OK.k$, $c = OL.l$. That is three angles suffice for the determination of the elements.

TO DETERMINE THE POSITION OF ANY POLE P.

Using notation of Fig. 69,

$$\frac{b}{k}\cot PB = \frac{a}{h}\sin CK = \frac{c}{l}\sin AK;$$

$$\frac{\sin CK}{\sin AK} = \frac{ch}{al};\ \frac{h}{l} = \frac{\sin AD}{\sin CD}\cdot\frac{\sin CK}{\sin AK},$$

$$\frac{k}{l} = \frac{\sin AD}{\sin AK}\cdot\frac{\tan OB}{\tan PB};\ \cot AK - \cot AC =$$

$$\frac{h}{l}(\cot AD - \cot AC);\ \cos PA = \sin PB$$

$$\cos AK;\ \cos PC = \sin PB\ \cos CK$$

$$\tan PB = \frac{l}{k}\frac{\sin AD}{\sin AK}\tan OB.$$

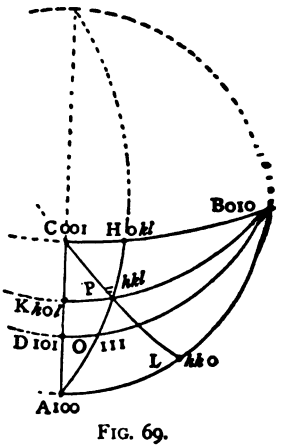


FIG. 69.

Because B is pole of zone circle AA', $CK = PBC$, $AK = PBA$ and $CK = 180° - (PBA + CA')$, that is the sines and cosines of these angles may be substituted for those of the arcs in any of the formulæ above.

To Find the Arc Joining two Poles, P and P'.

Proceed as in triclinic, p. 30.

ORTHORHOMBIC SYSTEM.

All forms in this system must be referable to three nonequivalent axes, Fig. 70, at right angles to each other. The three axes are physically of equal importance, any one may be chosen as c, the vertical; the longer of the other two will be the macro or b axis; the shorter axis (from front to back) the brachy or a axis. There are as many series of forms possible as there are irrational values for $\frac{a}{b}$ and $\frac{c}{b}$.

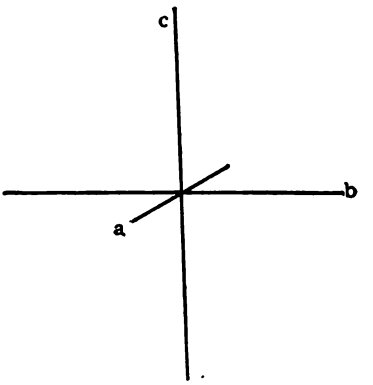


Fig. 70.

6. Class of the Rhombic Bisphenoid.

With three axes of binary symmetry at right angles to each other. As shown in the projection, Fig. 71, four faces satisfy the symmetry of the most general form or Rhombic Bisphenoid, Fig. 72. Example.—Epsomite.

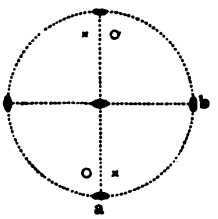


Fig. 71.

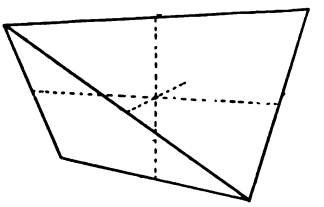


Fig. 72.

7. Class of the Rhombic Pyramid.

With two planes of symmetry perpendicular to each other and intersecting in an axis of binary symmetry. As shown in the projection, Fig. 73, four faces satisfy the symmetry of the most general form or Rhombic Pyramid, Fig. 74. Examples.—Calamine, struvite.

* Story-Maskelyne's *Crystallography*, p. 436 and Groth's *Phys. Kryst.*, p. 578.

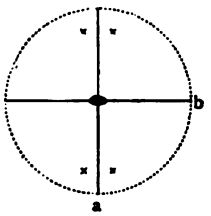


FIG. 73.

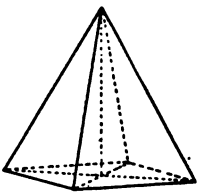


FIG. 74.

8. CLASS OF THE RHOMBIC BIPYRAMID.

With three planes of symmetry at right angles to each other which intersect in three axes of binary symmetry. These are shown in Fig. 75, and the planes divide space into eight octants, shown in projection, Fig. 76, as four trirectangular spherical triangles.

Any upper face corresponding to a pole × in the projection would, by rotation of 180° around the vertical binary axis, coincide with an upper face in the alternate octant, these reflected in the vertical symmetry planes coincide with two other upper faces and the four reflected in the horizontal plane coincide with four lower planes the poles of which are marked by circles. Since *h*, *k* and *l* retain a constant order, there can be sign permutations only corresponding to one plane in each octant.

The symmetry of the class is therefore satisfied by eight faces for the most general form or RHOMBIC BIPYRAMID, Fig. 77.* EXAMPLES.—Aragonite, marcasite, barite.

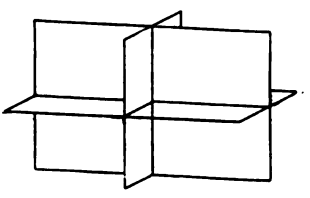


FIG. 75.

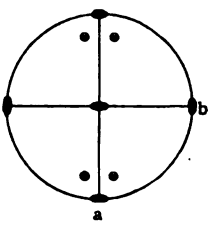


FIG. 76.

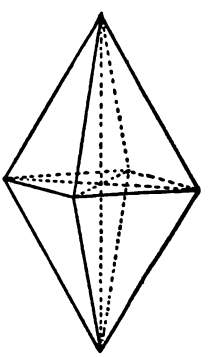


FIG. 77.

*There may be many different pyramids in a series with rational indices *hkl* which may be all equal, any two equal or all unequal.

The Six Limit Forms.

In each class there are six limit forms corresponding to special positions of the faces of the general form. These may be tabulated as follows:

Position of Any Face and its Pole.	Symbol*	Name of Form.	Classes to which Form Belongs.
1. Parallel to *a* and *b*. . . . . Poles are projected at center.	{001}	Basal Pinacoid, Fig. 78. Basal Plane. one face of Fig 78.	6, —, 8. —, 7, —.
2. Parallel to *a* and *c*. . . . . Poles are at intersections of *b* axis and primitive circle	{010}	Pinacoid, Fig. 79. (Brachy Pinacoid).	6, 7, 8.
3. Parallel to *b* and *c* . . . . Poles are at intersections of *a* axis and primitive circle	{100}	Pinacoid, Fig. 80. (Macro Pinacoid).	6, 7, 8.
4. Parallel to *a*. Poles are on the *b* axis	{0*kl*}	Prism, Fig. 81. (Brachy Dome). Dome, Fig. 84.	6, —, 8. —, 7, —.
5. Parallel to *b*. Poles are on the *a* axis	{*h*0*l*}	Prism, Fig. 82. (Macro Dome). Dome, Fig. 85.	6, —, 8. —, 7, —.
6. Parallel to *c*. Poles are on the primitive circle . . .	{*hk*0}	Prism, Fig. 83. (Rhombic Prism).	6, 7, 8.

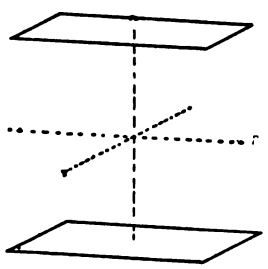


Fig. 78.

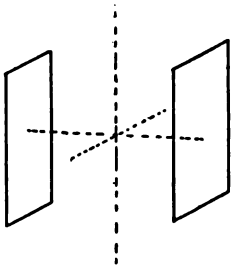


Fig. 79.

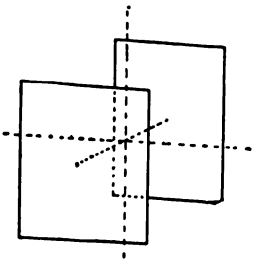


Fig. 80.

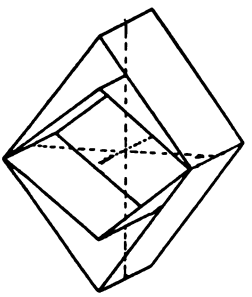


Fig. 81.

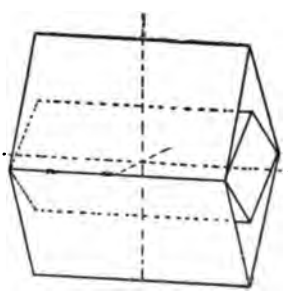


Fig 82.

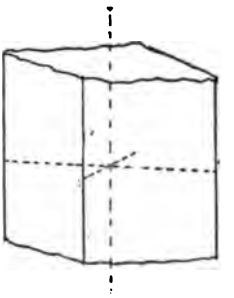


Fig. 83.

Limit Forms of Class 8.

* To obtain type symbols. The order is invariably *hkl* with reference to *a*, *b*, *c*. Any may become zero.

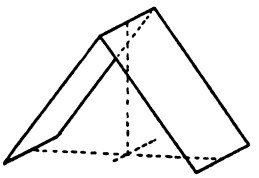


FIG. 84.

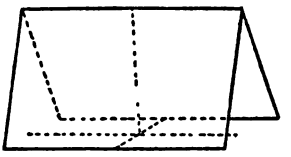


FIG. 85.

OTHER LIMIT FORMS WHICH ARE NEW SHAPES.

PROJECTION AND CALCULATION OF ORTHORHOMBIC FORMS.

The pinacoid (001) is usually selected as the plane of projection. The poles are as in the table and their exact positions usually result from the intersections of known zones or by Problem 3, p. 22. It is sometimes convenient to calculate the position of a plane (*hk*0) corresponding to the plane (*hkl*), lay off this on the primitive circle thus determining the zone [*hk*0 001].

CALCULATION OF ELEMENTS.

The interaxial angles are all right angles.

In the spherical triangle *rst*, $t = 90°$, $r = hkl : 010$, $s = hkl : 100$
Substituting in formulæ p. 34.

$$\cos tr = \frac{\cos s}{\sin r}, \quad \cos ts = \frac{\cos r}{\sin s}, \quad \frac{OL}{OK} = \cot ts, \quad \frac{OL}{OH} = \cot tr$$

$$a = OH.h, \quad b = OK.k, \quad c = OL.l.$$

Also, $a = \tan \frac{1}{2} (100 : \bar{1}10)$ and $c = \tan \frac{1}{2} (011 : 0\bar{1}1)$

$$= a \tan \frac{1}{2} (101 : \bar{1}01).$$

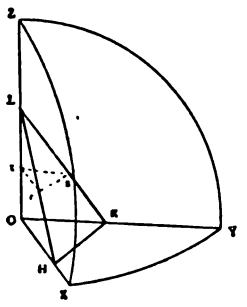


FIG. 86.

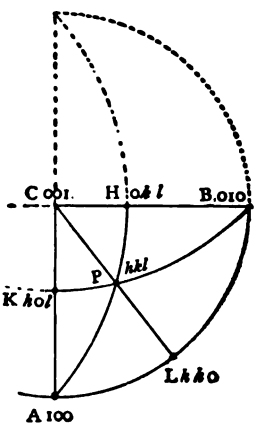


FIG. 87.

GENERAL EQUATION BETWEEN AXES AND INDICES.

$$\frac{a}{h}\cos PA = \frac{b}{k}\cos PB, = \frac{c}{l}\cos PC.$$

TO DETERMINE* ANY POLE P.

$$\cot PA = \frac{hb}{ka}\cos PAB = \frac{hc}{la}\cos PAC = \frac{bc}{a}\frac{h}{\sqrt{c^2k^2+b^2l^2}}$$

$$\cot PB = \frac{kc}{lb}\cos PBC = \frac{ka}{hb}\cos PBA = \frac{ca}{b}\frac{k}{\sqrt{a^2l^2+c^2h^2}}$$

$$\cot PC = \frac{la}{hc}\cos PCA = \frac{lb}{kc}\cos PCB = \frac{ab}{c}\frac{l}{\sqrt{b^2h^2+a^2k^2}}$$

$$\tan PCA = \frac{ka}{bh},\ \tan PBC = \frac{hc}{la},\ \tan PAB = \frac{lb}{kc}$$

For Unit Plane O

$$a : b : c = \cos OB \cos OC : \cos OC \cos OA : \cos OA \cos OB.$$

TO FIND THE ARC JOINING ANY TWO POLES P and P'

Let S denote $h^2b^2c^2 + k^2c^2a^2 + l^2a^2b^2$, S' a similar quantity from indices $h'k'l'$ of second pole.

$$\cos PP' = \frac{hh'b^2c^2 + kk'c^2a^2 + ll'a^2b^2}{\sqrt{SS'}}$$

If $P' = A, B, C, H, K$ or L of Fig. 87

$$\cos PA = \sin PH = \frac{hbc}{\sqrt{S}},\quad \cos PB = \sin PK = \frac{kca}{\sqrt{S}},$$

$$\cos PC = \sin PL = \frac{lab}{\sqrt{S}}$$

If P and P' are faces of the same form †

$$\tan PCA = \frac{k}{h}a;\ \tan \tfrac{1}{2}(hkl) : (\bar{h}kl) = \frac{a}{c}\cdot\frac{l}{h}\cos PCA$$

$$\sin \tfrac{1}{2}(hkl) : (\bar{h}kl) = \cos \tfrac{1}{2}(hkl) : (hkl)\cdot\cos PCA$$

$$\sin \tfrac{1}{2}(hkl) : (\bar{h}kl) = \cos \tfrac{1}{2}(hkl) : (hkl)\sin PCA.$$

TANGENT RELATION BETWEEN $hkl = P$, and $h'k'l = P'$ which lie in a zone with A, B or C.

$$\frac{h'}{h}\cdot\frac{\tan P'A}{\tan PA} = \frac{k'}{k} = \frac{l'}{l};\ \frac{k'}{k}\cdot\frac{\tan P'B}{\tan PB} = \frac{l'}{l} = \frac{h'}{h};\ \frac{l'}{l}\cdot\frac{\tan P'C}{\tan PC} = \frac{h'}{h} = \frac{k'}{k}$$

* Miller's *Crystallography*, p. 79, and Story-Maskelyne's *Crystallography*, p. 443.
† Groth, *Phys. Kryst.*, p. 574.

TETRAGONAL SYSTEM.

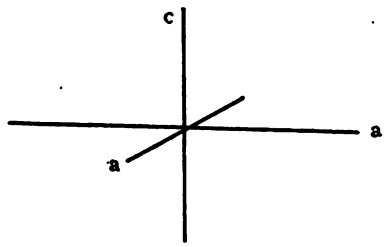


FIG. 88.

All forms of this system must be referable to two equivalent axes, a, at 90° to each other, Fig. 88, and the axis c, conventionally vertical, at 90° to both.

The forms possible on crystals of the same substance can all be referred to one value of $\frac{c}{a}$.

9. CLASS OF THE THIRD ORDER BISPHENOID.

With composite symmetry to a quaternary axis and a plane at right angles thereto. As shown in the projection, Fig. 89, four faces satisfy the symmetry for the most general form or TETRAGONAL BISPHENOID OF THIRD ORDER, Fig. 90. No examples are known.

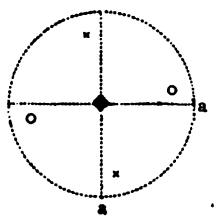


FIG. 89.

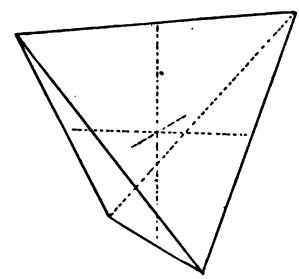


FIG. 90.

10. CLASS OF THE TETRAGONAL PYRAMID OF THIRD ORDER.

With one axis of quaternary symmetry. As shown in the stereographic projection, Fig. 91, four faces satisfy the symmetry for the most general form or TETRAGONAL PYRAMID OF THIRD ORDER, Fig. 92. EXAMPLE.—Wulfenite.

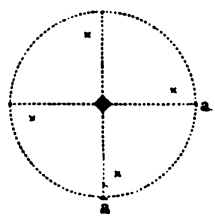


FIG. 91.

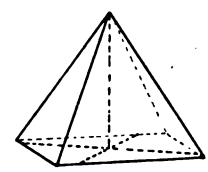


FIG. 92.

11. SCALENOHEDRAL CLASS.

With two planes of symmetry at right angles to each other and intersecting in an axis of quaternary symmetry. Also two axes of binary symmetry midway between the planes. As shown in the projection, Fig. 93, eight faces satisfy the symmetry for the most general form or SCALENOHEDRON, Fig. 94. EXAMPLE.—Chalcopyrite.

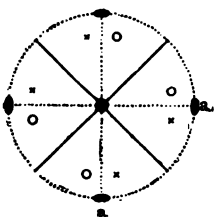


FIG. 93.

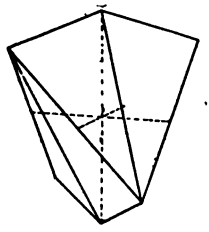


FIG. 94.

12. TRAPEZOHEDRAL CLASS.

Without planes of symmetry, but with the five axes of Class 15. As shown in the projection, Fig. 95, eight faces satisfy the symmetry for the most general form or TRAPEZOHEDRON, Fig. 96. EXAMPLE.—Nickel sulphate, $NiSO_4.6H_2O$.

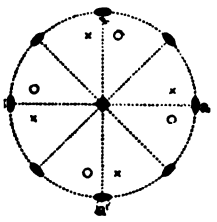


FIG. 95.

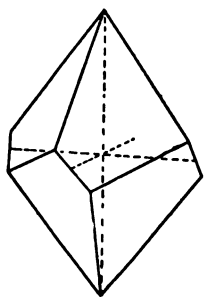


FIG. 96.

13. CLASS OF THE TETRAGONAL BIPYRAMID OF THIRD ORDER.

With one horizontal plane of symmetry and one vertical axis of quaternary symmetry. As shown in the projection, Fig. 97, eight faces satisfy the symmetry for the most general form or TETRAGONAL BIPYRAMID OF THIRD ORDER, Fig. 98. EXAMPLE.—Scheelite.

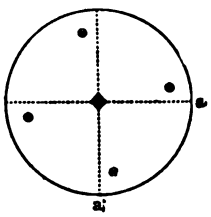


FIG. 97.

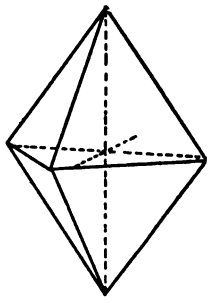


FIG. 98.

14. CLASS OF THE DITETRAGONAL PYRAMID.

With four vertical planes of symmetry intersecting in an axis of quaternary symmetry. As shown in the projection, Fig. 99, the essential change from Class 15 is the omission of the plane of symmetry normal to the quaternary axis. The general form is therefore geometrically like the upper or lower half of Fig. 103. Fig. 100 shows the six-faced most general form or DITETRAGONAL PYRAMID. EXAMPLE.—Silver fluoride, $AgF.H_2O$.

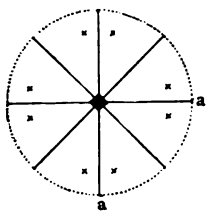


FIG. 99.

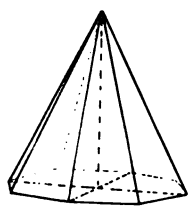


FIG. 100.

15. CLASS OF THE DITETRAGONAL BIPYRAMID.

With four planes of symmetry at 45° to each other, which intersect in an axis of quaternary symmetry, and one plane normal to these which intersects the four planes in axes of binary symmetry.

The planes divide space into sixteen sections, Fig. 101 shown in the projection, Fig. 102, as eight birectangular spherical triangles with angles at the center of 45°. Any upper face corresponding to a pole ×, by rotations of 90° coincides successively with three other upper faces. These four reflected in a vertical planes coincide with four others and the eight reflected in a horizontal plane coincide with eight lower faces marked with a circle.

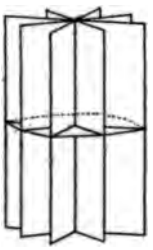


FIG. 101.

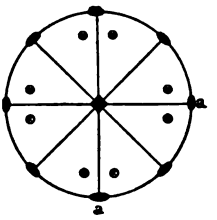


FIG. 102.

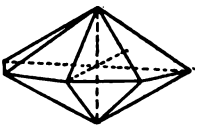


FIG. 103.

Since *l* remains with the third axis, the letter permutations are only *hkl* and *khl*. Each of these is subject to eight permutations in sign $+++, +-+, --+, -++, ++-, +--, ---, -+-$.

Both symmetry and permutations show that there must be in this class sixteen faces in the most general form or DITETRAGONAL BIPYRAMID Fig. 103. EXAMPLES.—Zircon, cassiterite, rutile.

LIMIT FORMS OF CLASS 15.

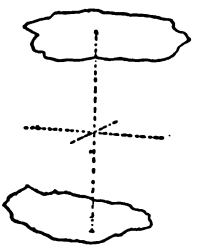


FIG. 104.

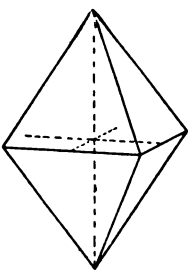


FIG. 105.

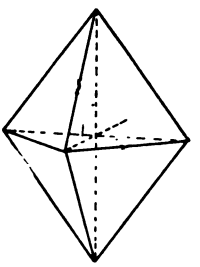


FIG. 106.

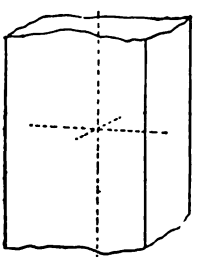


FIG. 107.

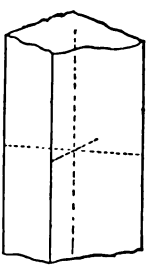


FIG. 108.

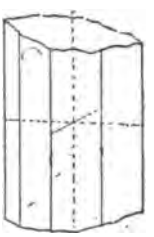


FIG. 109.

The Six Limit Forms.

In each class there are six limit forms corresponding to special positions of the faces of the general form. These may be tabulated as follows:

Position of any Face and its Pole.	Symbol*	Name of Form.	Classes to which the Form belongs.
1. Intersects the vertical axis and is parallel to both basal axes. Poles are projected at the center.	{001}	Basal Pinacoid, Fig. 104.	15, —, 13, 12, 11, —, 9.
		Basal Plane, one face of Fig. 104.	—, 14, —, —, —, 10, —.
2. Intersects the vertical axis and is parallel to one basal axis. Poles are on axial diameters.	{*h*0*l*}	Tetragonal Bipyramid, Second Order, Fig. 105.	15, —, 13, 12, 11, —, —.
		Tetragonal Pyramid, Second Order, Fig. 113.	—, 14, —, —, —, 10, —.
		Tetragonal Bisphenoid, Second Order, Fig. 111.	—, —, —, —, —, —, 9.
3. Intersects the vertical axis and is equally inclined to both basal axes. Poles are on diagonal diameters.	{*hhl*}	Tetragonal Bipyramid, First Order, Fig. 106.	15, —, 13, 12, —, —, —.
		Tetragonal Pyramid, First Order, Fig. 114.	—, 14, —, —, —, 10, —.
		Tetragonal Bisphenoid, First Order, Fig. 110.	—, —, —, —, 11, —, 9.
4. Parallel to the vertical axis and to one basal axis. Poles are at intersections of primitive circle and axial diameters.	{100}	Tetragonal Prism, Second Order, Fig. 107.	15, 14, 13, 12, 11, 10, 9.
5. Parallel to the vertical axis and equally inclined to the basal axes. Poles are at intersections of primitive circle and diagonal diameters.	{110}	Tetragonal Prism, First Order, Fig. 108.	15, 14, 13, 12, 11, 10, 9.
6. Parallel to the vertical axis and unequally inclined to the basal axes. Poles are on the primitive circle.	{*hk*0}	Ditetragonal Prism, Fig. 109.	15, 14, —, 12, 11, —, —.
		Tetragonal Prism, Third Order, Fig. 112.	—, —, 13, —, —, 10, 9.

* The first and second indices will be *h* and *k* or both *h* if equal, or *ho* if the face is parallel to one basal axis. The third symbol will be *i* or *o* as the face intersects or is parallel to the vertical axis. These must be reduced to simplest form.

LIMIT FORMS OF CLASS 15.

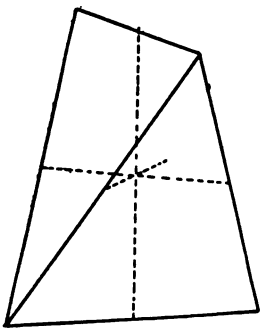


FIG. 110.

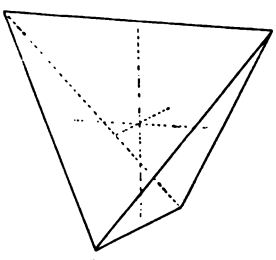


FIG. 111.

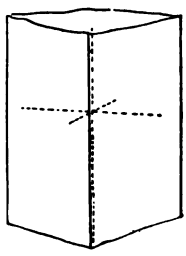


FIG. 112.

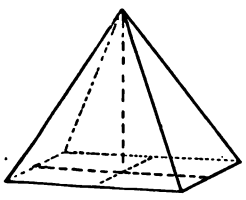


FIG. 113.

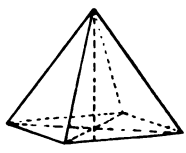


FIG. 114.

OTHER LIMIT FORMS WHICH ARE NEW SHAPES.

PROJECTION AND CALCULATION OF TETRAGONAL FORMS.

The basal pinacoid is usually selected as the plane of projection, the poles lying as stated in the table. On p. 23, Fig. 36, is described the projection of the poles of a crystal of cassiterite.

DETERMINATION OF ELEMENTS.

In Fig. 86, $\cos tr = \frac{\cos s}{\sin r}$ and $\frac{\text{OL}}{\text{OH}} = \cot tr$. $c = \text{OL}.l$

Also in Fig. 115 if hhl = F.

$$c = \tan \text{FC} \cos 45° = \tan \text{KC}$$

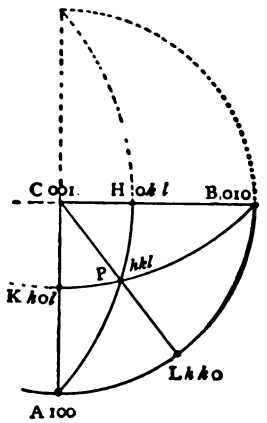
$$\sin \text{KC} = \tan \text{KF} \cot 45$$

$$c = \frac{l}{\sqrt{h^2+k^2}} \tan \text{PC}$$

TO DETERMINE POSITION OF ANY POLE* P.

$$\cot \text{PA} = \tan \text{PH} = \frac{h}{k} \cos \text{PAB}$$

$$= \frac{hc}{la} \cos \text{PAC} = \frac{ch}{\sqrt{c^2k^2+a^2l^2}}$$



* Miller's *Crystallography*, p. 44, Story-Maskelyne's *Crystallography*, p. 449.

$$\cot \text{PB} = \tan \text{PK} = \frac{k}{h} \cos \text{PBA} = \frac{kc}{la} \cos \text{PBC} = \frac{kc}{\sqrt{a^2l^2 + c^2h^2}}$$

$$\cot \text{PC} = \tan \text{PL} = \frac{la}{hc} \cos \text{PCA} = \frac{la}{kc} \cos \text{PCB} = \frac{la}{\sqrt{c^2(h^2 + k^2)}}$$

General Equation Between Axes and Indices.

$$\frac{a}{h} \cos PA = \frac{a}{k} \cos PB = \frac{c}{l} \cos PC.$$

Formula for Angle Between Two Planes.

$$\cos PP' = \frac{c^2(hh' + kk') + a^2ll'}{\sqrt{c^2(h^2 + k^2) + a^2l^2}\ \sqrt{c^2(h'^2 + k'^2) + a^2l'^2}}$$

Formulæ for angle between two faces of same form are as on p. 39.

Angle $P'C$ When PC Known

$$\frac{l'}{\sqrt{h'^2 + k'^2}} \tan P'C = \frac{l}{\sqrt{h^2 + k^2}} \tan PC.$$

HEXAGONAL SYSTEM.

All forms in this system must be referable to three equivalent axes a in one plane at 60° to each other and a fourth axis c normal to these, conventionally placed vertically. Fig. 116.

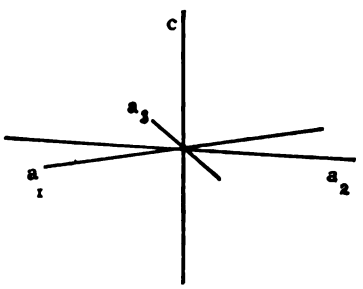


Fig. 116.

The system comprises two grand divisions. The Hexagonal division with five classes in which the fourth axis is an axis of senary symmetry. The Rhombohedral division with seven classes in which this axis is an axis of ternary symmetry.

The vertical and basal axes being non-equivalent, the ratio of the parameters $\frac{c}{a}$ is always an irrational number and for each different ratio a different series of forms exists, one series only being capable of occurrence on the crystals of the same substance.

The basal axes are most conveniently considered in the order of

the figure, for then* the third index is always the algebraic sum of the first and second.

THE RHOMBOHEDRAL DIVISION.

16. Class of the Trigonal Pyramid of Third Order.

Without planes of symmetry, but with an axis of ternary symmetry. As shown in projection, Fig. 117, the symmetry is satisfied by three faces for the general form, or Trigonal Pyramid of Third Order, Fig. 118. Examples.—Sodium periodate $NaIO_4.3H_2O$.

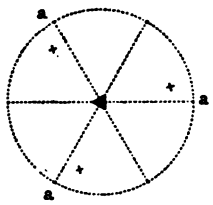


Fig. 117.

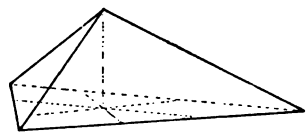


Fig. 118.

17. Class of Rhombohedron of the Third Order.

With composite symmetry to a ternary axis and a plane at right angles thereto. As shown in the stereographic projection, Fig. 119, the symmetry is satisfied by six faces for the most general form or Rhombohedron of the Third Order, Fig. 120. Examples.—Dioptase, phenacite.

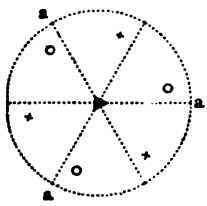


Fig. 119.

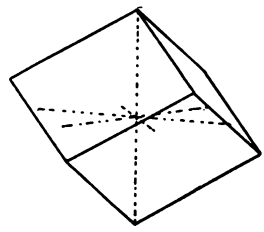


Fig. 120.

18. Class of the Trigonal Trapezohedron.

Without planes of symmetry, but with the four axes of symmetry of Class 22. As shown in projection, Fig. 121, the symmetry is satisfied by six planes for the general form, or Trigonal Trapezohedron, Fig. 122. Examples.—Quartz, cinnabar.

* Simple proof in Bauerman's Systematic Mineralogy, p. 76.

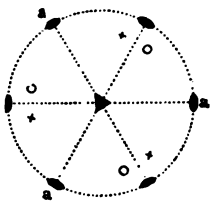


FIG. 121.

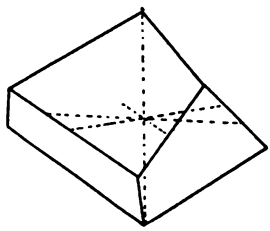


FIG. 122.

19. CLASS OF THE TRIGONAL BIPYRAMID OF THIRD ORDER.

With one plane of symmetry at right angles to the axis of ternary symmetry. As shown in the projection, Fig. 123, the symmetry is satisfied by six planes for the most general form, or TRIGONAL BIPYRAMID OF THIRD ORDER, Fig. 124. No examples are known.

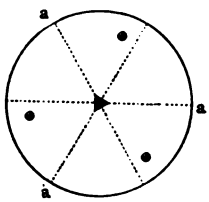


FIG. 123.

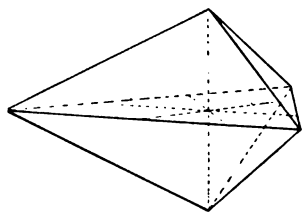


FIG. 124.

20. CLASS OF THE DITRIGONAL PYRAMID.

With three planes of symmetry at 60° to each other which intersect in the axis of ternary symmetry. These forms are geometrically the upper or lower halves of those of Class 22. As shown in projection, Fig. 125, the symmetry is satisfied by six planes for the most general form, or DITRIGONAL PYRAMID, Fig. 126. EXAMPLES.—Tourmaline, proustite, pyrargyrite.

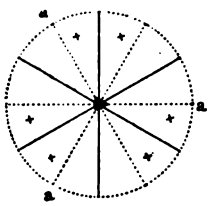


FIG. 125.

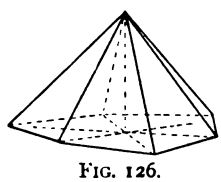


FIG. 126.

21. CLASS OF THE DITRIGONAL SCALENOHEDRON.

With three planes of symmetry at 60° to each other and intersecting in an axis of ternary symmetry, and three axes of binary

symmetry bisecting the angles between the planes. As shown in projection, Fig. 127, the symmetry is satisfied by twelve faces for the most general form, or DITRIGONAL SCALENOHEDRON, Fig. 128. EXAMPLES.—Calcite, hematite, corundum.

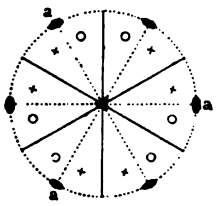


FIG. 127.

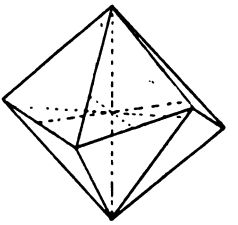


FIG. 128.

22. CLASS OF THE DITRIGONAL BIPYRAMID.

With three planes of symmetry at 60° to each other which intersect in an axis of ternary symmetry and one plane normal to these which intersects the others in axes of binary symmetry. The planes, Fig. 129, divide space into twelve equal parts, shown in projection, Fig. 130, as six birectangular spherical triangles with angles at the center 60°.

Any upper face, corresponding to a pole × in one of the triangles, would by rotations of 120° around the ternary axis coincide with two other upper faces. Each of these by reflection in a vertical plane of symmetry coincides with another face and the six by reflection in the horizontal plane of symmetry coincide with six lower faces, the poles of which are indicated by circles.

The symmetry of the class is satisfied by twelve faces for the most general form or DITRIGONAL BIPYRAMID, Fig. 131. No examples are known.

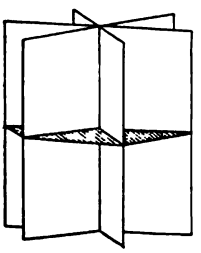


FIG. 129.

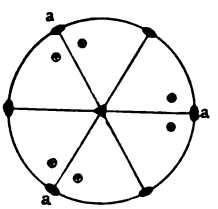


FIG. 130.

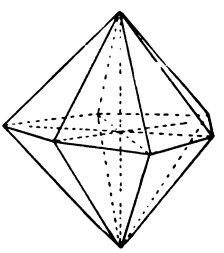


FIG. 131.

The Six Limit Forms.

In each class there are six limit forms corresponding to special positions of the faces of the general form. These may be tabulated as follows:

Position of any Face and its Pole.	Symbol*	Name of Form.	Classes to which the form belongs.
1. Parallel to all basal axes. Poles projected at the centre.	$\{0001\}$	Basal Pinacoid, Fig. 153.	22, 21, —, 19, 18, 17, —.
		Basal Plane, one plane of Fig. 153.	—, —, 20, —, —, —, 16.
2. Oblique to the vertical axis but parallel to one basal axis. Poles are on the interaxial diameters.	$\{h0\bar{h}i\}$	Trigonal Bipyramid First Order, Fig. 132.	22, —, —, 19, —, —, —.
		Rhombohedron First Order, Fig. 135.	—, 21, —, —, 18, 17, —.
		Trigonal Pyramid First Order, Fig. 136.	—, —, 20, —, —, —, 16.
3. Oblique to the vertical axis and equally inclined to alternate basal axes. Poles are on the axes.	$\{hh2hi\}$	Hexagonal Bipyramid Second Order, Fig. 155.	22, 21, —, —, —, —, —.
		Hexagonal Pyramid Second Order, Fig. 160.	—, —, 20, —, —, —, —.
		Trigonal Bipyramid Second Order, Fig. 137.	—, —, —, 19, 18, —, —.
		Rhombohedron Second Order, Fig. 140.	—, —, —, —, —, 17, —.
		Trigonal Pyramid Second Order, Fig. 141.	—, —, —, —, —, —, 16.
4. Parallel to the vertical axis and to one basal axis. Poles are at intersections of primitive circle and interaxial diameters.	$\{10\bar{1}0\}$	Trigonal Prism First Order, Fig. 133.	22, 21, 20, 19, —, —, 16.
		Hexagonal Prism First Order, Fig. 156.	—, —, —, —, 18, 17, —.
5. Parallel to the vertical axis and equally inclined to alternate basal axes. Poles are at intersections of axes and primitive circle.	$\{11\bar{2}0\}$	Hexagonal Prism Second Order, Fig. 157.	22, 21, 20, —, 18, 17, —.
		Trigonal Prism Second Order, Fig. 138.	—, —, —, 19, —, —, 16.
6. Parallel to the vertical and unequally inclined to all basal axes. Poles are on primitive circle.	$\{hk\bar{i}0\}$	Ditrigonal Prism, Fig. 134.	22, —, 20, —, 81, —, —.
		Dihexagonal Prism, Fig. 158.	—, 21, —, —, —, —, —.
		Trigonal Prism Third Order, Fig. 139.	—, —, —, 19, —, —, 16.
		Hexagonal Prism Third Order, Fig. 161.	—, —, —, —, —, 17, —.

* See footnote under hexagonal division, p. 54.

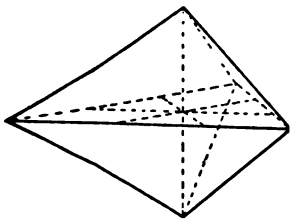


FIG. 132.

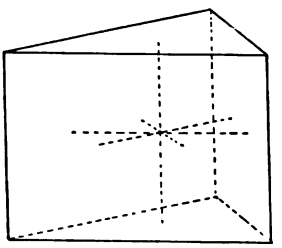


FIG. 133.

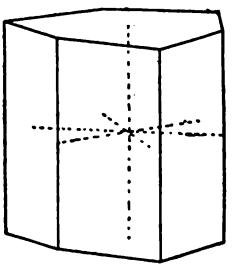


FIG. 134.

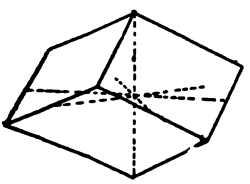


FIG. 135.

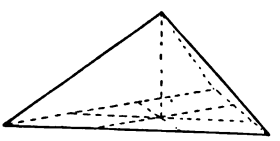


FIG. 136.

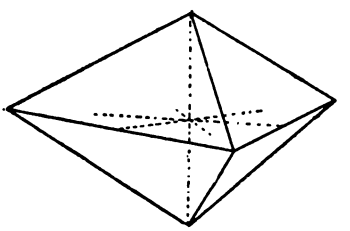


FIG. 137.

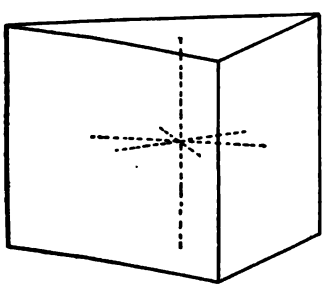


FIG. 138.

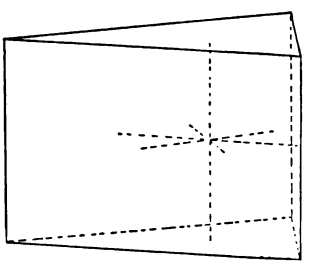


FIG. 139.

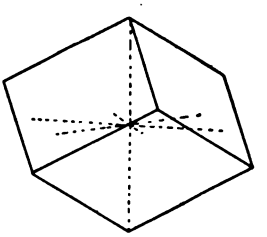


FIG. 140.

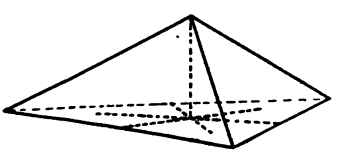


FIG. 141.

LIMIT FORMS IN RHOMBOHEDRAL DIVISION WHICH ARE GEOMETRICALLY SHAPES NOT INCLUDED IN HEXAGONAL DIVISION.

THE HEXAGONAL DIVISION.

23. Class of the Third Order Hexagonal Pyramid.

Without planes of symmetry but with one axis of senary symmetry. As shown in the projection, Fig. 142, six planes satisfy the symmetry of the most general form or Hexagonal Pyramid of Third Order, Fig. 143. Example.—Nephelite.

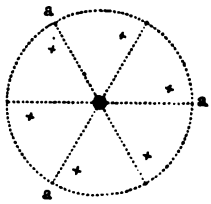


Fig. 142.

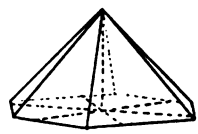


Fig. 143.

24. Class of the Hexagonal Trapezohedron.

Without planes of symmetry but with the seven axes of Class 27. As shown in projection, Fig. 144, twelve faces satisfy the symmetry for the most general form or Hexagonal Trapezohedron, Fig. 145. Example.—$(SbO)_2BaC_4H_4O_6.KNO_3$.

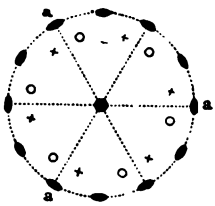


Fig. 144.

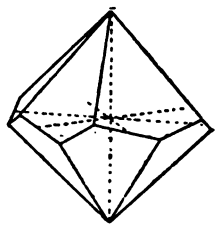


Fig. 145.

25. Class of the Third Order Hexagonal Bipyramid.

With one plane of symmetry normal to the axis of senary symmetry. As shown in projection, Fig. 146, twelve faces satisfy the symmetry for the most general form or Hexagonal Bipyramid of Third Order, Fig. 147. Example.—Apatite, pyromorphite.

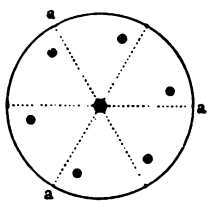


Fig. 146.

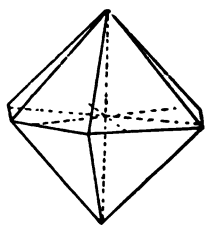


Fig. 147.

26. Class of the Dihexagonal Pyramid.

With six planes of symmetry at 30° to each other and intersecting in the axis of senary symmetry. From the projection, Fig. 148, it is evident that the general form, or Dihexagonal Pyramid, Fig. 149, is geometrically the upper (or lower) half of the general form, Fig. 152, of the next class.

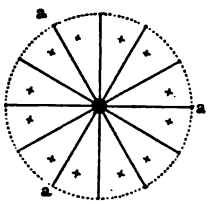


Fig. 148.

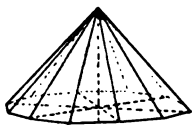


Fig. 149.

27. Class of the Dehexagonal Bipyramid.

With seven planes of symmetry, six being normal to the seventh and at 30° to each other as shown in Fig. 150. The common intersection of six planes of symmetry is the axis of senary symmetry and their intersections with the seventh are axes of binary symmetry.

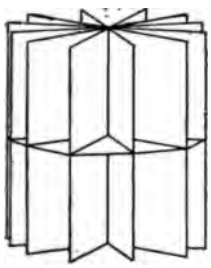


Fig. 150.

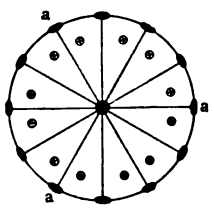


Fig. 151.

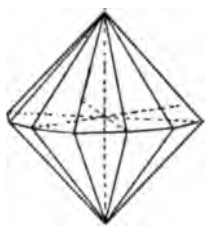


Fig. 152.

In projection, Fig. 151, the planes of symmetry form twelve equal birectangular spherical triangles with angles at the centre, 30°. Any upper face, with a pole marked ×, by rotations of 60° around the senary axis coincides successively with five other upper faces represented by crosses in the alternate triangles. Each of the six reflected in a vertical plane of symmetry coincides with another face and the twelve reflected in the horizontal plane coincide with twelve lower faces represented by circles.

The letter permutations of the general symbol ($hk\bar{i}l$), in which i always must remain in fourth place, are *hkli*, *hlki*, *khli*, *klhi*, *lhki*, *lkhi*, and each may have four sign permutations, $++-+$, $--++$, $++--$ and $--+-$ or twenty-four permutations in all. Projection and permutation both show that the symmetry of the class is satisfied by twenty-four faces for the most general form or DIHEXAGONAL BIPYRAMID, Fig. 152. EXAMPLE.—Beryl.

THE SIX LIMIT FORMS.

In each class there are six limit forms corresponding to special positions of the faces of the general form. These may be tabulated a follows:

Position of any Face and its Pole.	Symbol*	Name of Form.	Classes to which the form bel'ng's
1. Parallel to all basal axes. Poles projected at the center.	$\{0001\}$	BASAL PINACOID, Fig. 153. BASAL PLANE, one plane of Fig. 153.	27,—,25,24,—. —,26,—,—,23.
2. Oblique to the vertical axis but parallel to one basal axis. Poles are on the interaxial diameters.	$\{h0\bar{h}i\}$	HEXAGONAL BIPYRAMID FIRST ORDER, Fig. 154. HEXAGONAL PYRAMID FIRST ORDER, Fig. 159.	27,—,25,24,—. —,26,—,—,23.
3. Oblique to the vertical axis and equally inclined to alternate basal axes. Poles are on the axes.	$\{hh2\bar{h}i\}$	HEXAGONAL BIPYRAMID SECOND ORDER, Fig. 155. HEXAGONAL PYRAMID SECOND ORDER, Fig. 160.	27,—,25,24,—. —,26,—,—,23.
4. Parallel to the vertical axis and to one basal axis. Poles are at intersections of primitive circle and interaxial diameters.	$\{10\bar{1}0\}$	HEXAGONAL PRISM FIRST ORDER, Fig. 156.	27,26,25,24,23.
5. Parallel to the vertical axis and equally inclined to alternate basal axes. Poles are at the intersections of axes and primitive circle.	$\{11\bar{2}0\}$	HEXAGONAL PRISM SECOND ORDER, Fig. 157.	27,26,25,24,23.
6. Parallel to the vertical but unequally inclined to all basal axes. Poles are on the primitive circle.	$\{hk\bar{i}0\}$	DIHEXAGONAL PRISM, Fig. 158. HEXAGONAL PRISM THIRD ORDER, Fig. 161.	27,26,—,24,—. —,—,25,—,23.

* In the type symbols the first and second indices relate to the relative inclination of the faces to the alternate basal axes. If unequally inclined *hk*, if equally inclined *hh*, if parallel to one *ho*. The third index is $\bar{i}$, the algebraic sum of the first and second with the opposite sign.

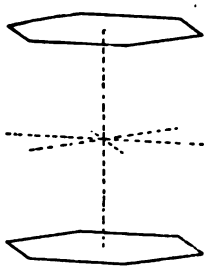


FIG. 153.

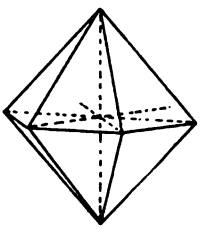


FIG. 154.

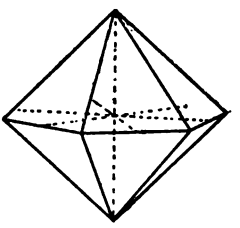


FIG. 155.

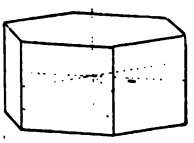


FIG. 156.

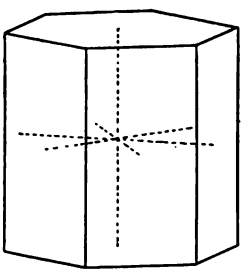


FIG. 157.

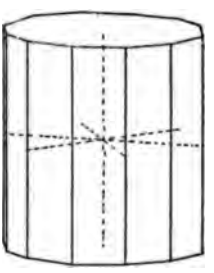


FIG. 158.

LIMIT FORMS OF CLASS 27.

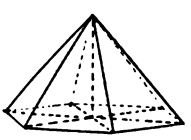


FIG. 159.

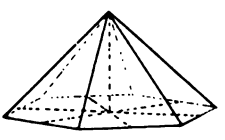


FIG. 160.

LIMIT FORMS GEOMETRICALLY NEW IN THE OTHER CLASSES.

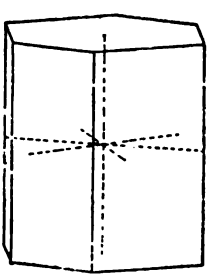


FIG. 161.

PROJECTION AND CALCULATION OF HEXAGONAL AND RHOMBOHEDRAL FORMS.

The basal pinacoid is the plane of projection. The position of the poles are as in the tables, pp. 50 and 54.

The indices of a face truncating an edge of any form are the sum of the indices of the faces forming the edge, for example, the edge between 1011 and 0111 is truncated by 1122.

THE ZONAL RELATIONS

Are calculated from three indices of which one must relate to the vertical axis. For instance, by method of cross multiplication

the zone indices of the planes $hk\bar{i}l, h'k'\bar{i}'l'$ will be determined by hkl and $h'k'l'$: $u = ki' - ik'$, $v = il' - hi'$, $w = hk' - kh'$.

DETERMINATION OF ELEMENTS.

Two equal basal axes at 120° to each other and at right angles to the vertical are sufficient. In Fig. 111 the spherical triangle rst has $t = 120°$.

$$\cos tr = \frac{\cos s + \frac{1}{2}\cos r}{\frac{1}{2}\sin r\sqrt{3}}; \cot tr = \frac{OL}{OH}; c = OL.i$$

or

$$\tan (0001):(1\bar{1}22) = \frac{h}{i} = \frac{c}{a} = c$$

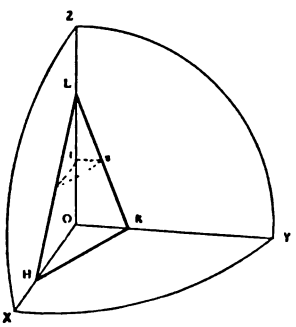


FIG. 162.

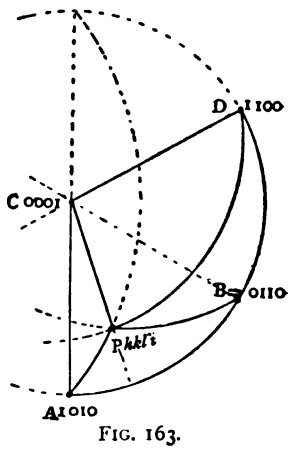


FIG. 163.

TO DETERMINE THE POSITION OF ANY POLE P.

Using notation of Fig. 163 and denoting by M the quantity

$$\sqrt{3i^2 + 4c^2(h^2 + k^2 + hk)}$$

$$\cos PA = \frac{c(2h + k)}{M}, \cos PB = \frac{c(h + 2k)}{M}, \cos PD = \frac{c(k - h)}{M}$$

$$\cos PC = \frac{i\sqrt{3}}{M}$$

TO FIND THE ARC JOINING TWO POLES P AND P'.

$$\cos PP' = \frac{3ii' + 2c^2(hk' + kh' + 2hh' + 2kk')}{MM'}$$

TANGENT PRINCIPLE.

$$\frac{i}{i'} \cdot \frac{\tan PC}{\tan P'C} = \frac{k}{k'} = \frac{h}{h'}$$

THE ISOMETRIC SYSTEM.

All forms in this system must be referrable to three equivalent axes at right angles to each other. The system includes five classes.

28. Class of the Tetartoid.

The forms have seven axes of symmetry but no planes of symmetry. Three of the axes are binary normal to the faces of the hexahedron and four are ternary through diagonally opposite solid angles of the hexahedron.

As shown in the projection Fig. 164, twelve faces in each alternate octant satisfy the symmetry for the most general form or Tetartoid, Fig. 165. Examples—Sodic chlorate, baric nitrate.

Fig. 164.

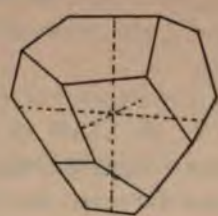


Fig. 165.

29. Class of the Gyroid.

Without planes of symmetry but with all the axes of symmetry of class 32. As shown in the projection Fig. 166, twenty-four faces, three in each octant, satisfy the symmetry for the most general form * or Gyroid, Figs. 167 and 168. Examples.—Halite, sylvite, cuprite.

Fig. 166.

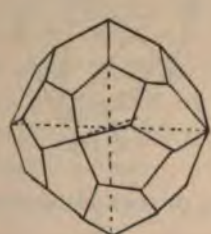


Fig. 167.

Fig. 168.

*The right form, Fig. 168, and the left form, Fig. 167, are enantiomorphic, that is, their elements are equal and the faces of the one are parallel but oppositely placed with respect to those of the other.

30. Class of the Diploid.

The forms have three cubic planes of symmetry, the intersection of these are three axes of binary symmetry and there are four diagonal axes of ternary symmetry.

As shown by the projection, Fig. 169, the symmetry of the class is satisfied by twenty-four faces for the most general form or Diploid, Figs. 170 and 171. Examples—Pyrite, smaltite, cobaltite.

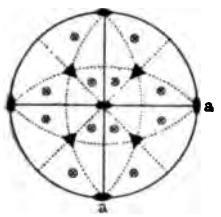


Fig. 169.

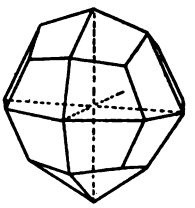


Fig. 170.

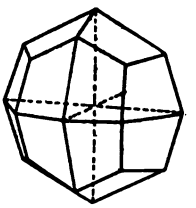


Fig. 171.

31. Class of the Hextetrahedron.

The forms * have the six dodecahedral planes of symmetry of Class 32 and the seven axes of symmetry formed by their intersection. Of the latter, four are ternary and three binary.

As shown in the projection, Fig. 172, the symmetry of this class is satisfied by twenty-four faces for the most general form or Hextetrahedron, Figs. 173 and 174. Examples.—Sphalerite, tetrahedrite, diamond.

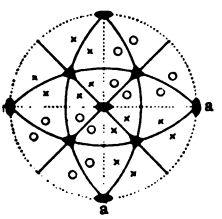


Fig. 172.

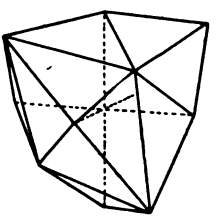


Fig. 173.

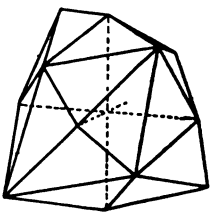


Fig. 174.

32. Class of the Hexoctahedron.

The forms of this class have three planes of symmetry parallel to the faces of the cube and six planes of symmetry through diag-

* The forms of this class exist in pairs which are said to be *congruent*, that is either by a revolution of 90° about an axis may brought into the position of the other. They are distinguished as right and left forms.

onally opposite edges of the cube, that is parallel to the faces of the rhombic dodecahedron Fig. 181.

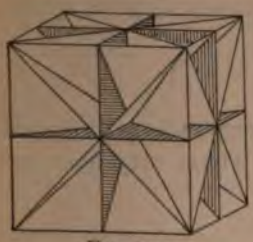


FIG. 175.

FIG. 176.

FIG. 177.

The intersections of these planes are axes of symmetry; three are quaternary the intersections of the planes parallel the hexahedron; four are ternary, each the intersections of three diagonal planes; and six are binary, each the intersection of a plane of each kind.

These planes and axes of symmetry are shown in stereographic projection, Fig. 176.

The number of faces in the most general form may be determined as follows:—

In any quadrant the pole marked 1 of a face in the upper half of the crystal by rotations of 120° around the ternary axis must coincide successively with faces the poles of which are 2 and 3. These three by reflection in the planes of symmetry must coincide 1 with 4, 2 with 5, 3 with 6. Since there is a vertical axis of quaternary symmetry the entire quadrant must coincide successively pole for pole with the second, third and fourth quadrants and finally the resulting 24 faces reflected in the horizontal plane of symmetry must coincide with 24 lower faces. That is, the form must consist of 48 faces with a pole in each of the right spherical triangles made by the planes of symmetry.

This result could also be reached by considering the possible permutations of letter and sign in the symbol of the general form $\{hkl\}$. The letter permutations are:

$$hkl,\ hlk,\ klh,\ khl,\ lkh,\ lhk.$$

Each of these can undergo eight permutations in sign according to the octant in which the plane occurs.

Upper octants $+\ +\ +,\ -\ +\ +,\ -\ -\ +,\ +\ -\ +,$
Lower octants $+\ +\ -,\ -\ +\ -,\ -\ -\ -,\ +\ -\ -,$

or 48 permutations in all. That is to satisfy the symmetry the most general form or HEXOCTAHEDRON, Fig. 177, must consist of 48 faces six in each octant. EXAMPLES.—Garnet, fluorite, spinel.

THE SIX LIMIT FORMS.

In each class there are six limit forms corresponding to special positions of the faces of the general form. These may be tabulated as follows:

Position of any Face.	Symbol*	Name of Form.	Classes to which Form belongs.
1. Equally inclined to all three axes. Poles on ternary axes.	{111}	OCTAHEDRON, Fig. 178.	32,—,30, 29,—.
		TETRAHEDRON, Fig. 184.	—,31,—,—, 28.
2. Equally inclined to two axes, more nearly parallel the third. Poles are on the short sides of triangles.	{*hhl*}	TRIGONAL TRISOCTAHEDRON, Fig. 179.	32,—,30, 29,—.
		TETRAGONAL TRISTETRAHEDRON, Fig. 185.	—,31,—,—,28.
3. Equally inclined to two axes less nearly parallel the third. Poles are on hypothenuse.	{*hkk*}	TETRAGONAL TRISOCTAHEDRON, Fig. 180.	32,—, 30,29,—.
		TRIGONAL TRISTETRAHEDRON, Fig. 186.	—, 31,—,—,28.
4. Equally inclined to two axes, parallel the third. Poles at vertices of right angles.	{110}	RHOMBIC DODECAHEDRON, Fig. 181.	32, 31,30,29,28.
5. Unequally inclined to two axes parallel the third. Poles are on the long sides.	{*hko*}	TETRAHEXAHEDRON, Fig. 182.	32,31,—,29,—.
		PENTAGONAL DODECAHEDRON, Fig. 187.	—,—, 30,—,28.
6. Parallel to two axes. Poles are on the quaternary axes.	{100}	HEXAHEDRON, 183.	32,31, 30,29,28.

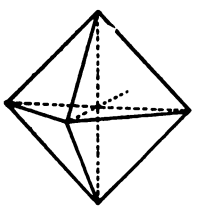


FIG. 178.

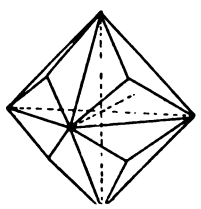


FIG. 179.

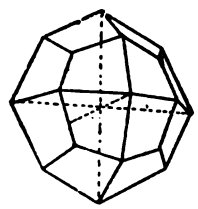


FIG. 180.

* The *type* symbols of the forms are easily obtained without direct reference to the intercepts by noting parallelism and relative nearness to parallelism to the axes. Zero being the symbol of parallelism and conventionally $h > k > l$.

FIG. 181.

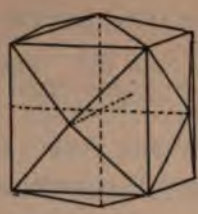


FIG. 182.

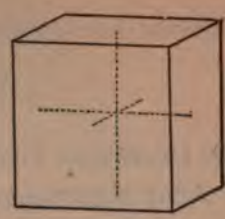


FIG. 183.

LIMIT FORMS OF CLASS 32.

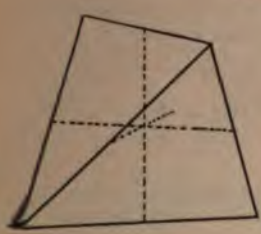


FIG. 184.

FIG. 185.

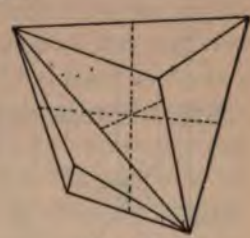


FIG. 186.

FIG. 187.

LIMIT FORMS GEOMETRICALLY NEW IN THE OTHER CLASSES.

PROJECTION AND CALCULATION* OF ISOMETRIC FORMS.

The forms are usually projected on a cubic face and the poles are as stated in the table, the positions being usually found by zone intersections. The planes of symmetry divide space into forty-eight similar parts each shown in stereographic projection as a right triangle in which: the hypothenuse (54° 44′), is the angle between a binary and a ternary axis; the longer side (45°) is the half angle between two binary axes; the shorter side (35° 16′) is the half angle between two ternary axes.

CALCULATION OF ELEMENTS.

The parameters are all equal and the angles between the axes are right angles.

*Millers' *Crystallography*, pp. 25; Story-Maskelyne's *Crystallography*, p. 453.

GENERAL EXPRESSION OF RELATION BETWEEN ANGLES AND INDICES.

$$\frac{\cos PA}{h} = \frac{\cos PB}{k} = \frac{\cos PC}{l}$$

TO DETERMINE POSITION OF ANY POLE. $P=(hkl)$.

Adopting notation of Fig. 188 equations become

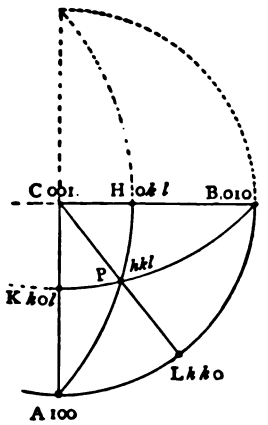


FIG. 188.

$$\cot CP = \frac{l}{\sqrt{h^2 + k^2}}, \tan ACP = \frac{k}{h},$$

$$\tan BAP = \frac{l}{k}; \tan = CBP\frac{h}{l}$$

$$\cos PA = \sin PH = \frac{h}{\sqrt{S}};$$

$$\cos PB = \sin PK = \frac{k}{\sqrt{S}}$$

$$\cos PC = \sin PL = \frac{l}{\sqrt{S}} \text{ in which}$$

$$S = h^2 + k^2 + l^2$$

TO FIND THE ARC JOINING $P = hkl$ and $P' = h'k'l'$

$$\cos PP' = \frac{hh' + kk' + ll'}{\sqrt{SS'}}$$

CHAPTER V.

MEASUREMENT OF CRYSTAL ANGLES.

The instruments used in measuring the angles between the faces of crystals are called *goniometers*, the simplest form being the application goniometer invented for Delisle by Carangeot and consisting of a semicircular graduated arc and two arms moving upon a pivot, the position of which may be changed according to the size and position of the crystal. In the older instruments the arms are fastened to the arc, but in the later types, as in Fig. 189, they are detached from the arc during measurement and replaced for the reading. In use the arms are each in close contact with a face and at right angles to the edge between the faces. They cannot be relied upon closer than one degree, and are, therefore, practically limited to the identification of previously measured angles.

FIG 189.

All measurements of accuracy are obtained by rotating the crystal around the edge between the faces and determining, by the aid of a ray of light *fixed in direction*, the angle between the position in which the first face gives a reflection and that in which the second face does the same.

In Fig. 190 the crystal is so adjusted that an edge coincides with the axis of

FIG. 190.

rotation *O*. The fixed direction of the ray of light is *CO1* that is, the incident ray is *CO* and the eye or telescope is at *T*. Then *OA* will be the direction of the first face when it acts as a reflecting surface, and *OB* will be the direction of the second face at that time, and only when rotated to the respective directions, *OA'* and *OB'*, will a reflection be obtained from the second face, that is, after a rotation measured by the equal arcs *AA'*, *BB'* or *NN'*, the normal angle between the faces.

Goniometers with Horizontal Axes.

The original reflection goniometer is that of Wollaston.* In this a base and column support in a collar a hollow axle to which there is attached at one end a vertical disc with a graduated rim and at the other end a handle. Through this axle passes a second axle, with at one end the crystal holder and at the other a handle. These two axles may be clamped to turn together, or, by the inner axle, the crystal may be rotated without the graduated circle.

In the simplest forms of this goniometer the crystal is fastened

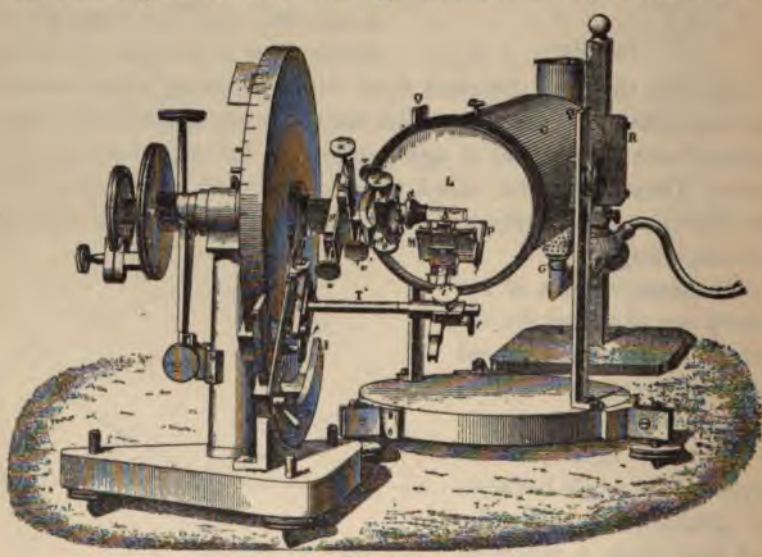


Fig. 191.

with wax to a thin, flexible, brass plate, and this is fixed in a holder which has several simple motions by means of which the

*W. H. Wollaston, Description of a reflective goniometer, *Phil. Trans.*, 1809, p. 253–259.

faces of the zone to be measured can be made to project clear of the apparatus and the edge to coincide with the axis of rotation. The signal may be a horizontal window bar twenty to thirty feet away, or better, a horizontal slit in a screen before an artificial light and it must be parallel to the axis of rotation. The eye is brought almost into contact with the crystal and there watches for the reflection of the signal as the faces successively move into position.

To secure a constant line of sight a reference mark may be made below the crystal parallel to the signal, or better, a second image of the signal may be obtained in a small mirror, the plane of which is parallel to the axis of rotation, or still better, a telescope with cross hairs may be used. In each case the rotation is continued until the image of the signal is bisected by or coincides with the reference mark.

Mallard's* modification of the Wollaston goniometer, shown in Fig. 191 on the left, differs from the earlier types in the substitution of a better crystal holder; the crystal is supported in the manner suggested by Groth, that is, it is fixed with wax on a small circular disc, d, and, by turning the screws v, v, and v', v', can receive two movements of rotation on two arcs of circles perpendicular to each other which have their common center near the middle of the crystal, A, so that the changes of orientation of the crystal do not too much displace its center of gravity.

For the proper centering, the entire system which holds the crystal is attached to two sliding planes, g and g', which, by means of the screws, u and u', impart to the crystal two movements of translation in a plane perpendicular to the axis of rotation. There is also a tangent screw for fine rotation and the mirror, M, has four motions of adjustment parallel to the axis of rotation. Any signal may be used, but preferably the collimator and artificial light, as shown in the figure.

The collimator is a cylinder, C, with at one end a large lens, L, and at the other, which is the exact focal plane of the lens, an adjustable plate, Fig. 192, pierced with signals, f, f', f'', of various forms beneath each of which is a fine reference slit. The light from an Argand gas lamp, R, or a Welsbach burner passes through the central signals, emerges as *parallel rays* from the lens and is reflected to the eye at the same time from the crystal face and the mirror.

* Er. Mallard, *Annales des Mines*, Nov.–Dec. 1887.

A greater degree of accuracy would be secured by the addition of an observation telescope, as in the Mitscherlich* modification of the Wollaston, but this is accompanied by loss of light and usually makes a dark room necessary, whereas the Mallard-Wollaston gives good results in ordinary light.

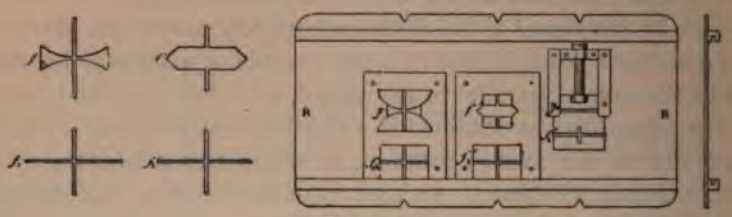


FIG. 192.

Professor Groth describes† a simple, inexpensive Wollaston with a telescope *fixed* parallel to the plane of the graduated circle and centered on the goniometer axis. The crystal holder is like that of the Mallard instrument, but the inner axle, which ordinarily rotates the crystal independently, is replaced by a screw which moves the crystal horizontally in the direction of the axis. The signal used by Professor Groth is a very small incandescent light at a distance of about thirty feet where it appears like a luminous point.

The principal objection to the Wollaston type of goniometer is that the weight of the crystal tends to throw it out of adjustment.

GONIOMETERS WITH VERTICAL AXES.

Babinet, v. Lang, Miller, Websky and others have gradually developed this more perfect and more generally applicable variety of goniometer, and there is little doubt that for simplicity of adjustments and perfection of construction the instrument known as the Fuess Model II. at present excels all others. It is shown in Fig. 193 and consists of:

a. THE STAND. A tripod with a conically bored head-piece supporting three concentric axles—*the outer axle* carrying the vernier circle and telescope, and turned by the latter; the fine adjustment is by the tangent screw, *F*, and the clamp screw, *a*,—*the middle axle* carrying the graduated circle and turned by the disc, *g*, or more

* Ueber ein Goniometer *Abhandl. Berlin Akad*, 1843, 189–197.

† *Physikalische Krystallographie*, III. Ed., p. 613.

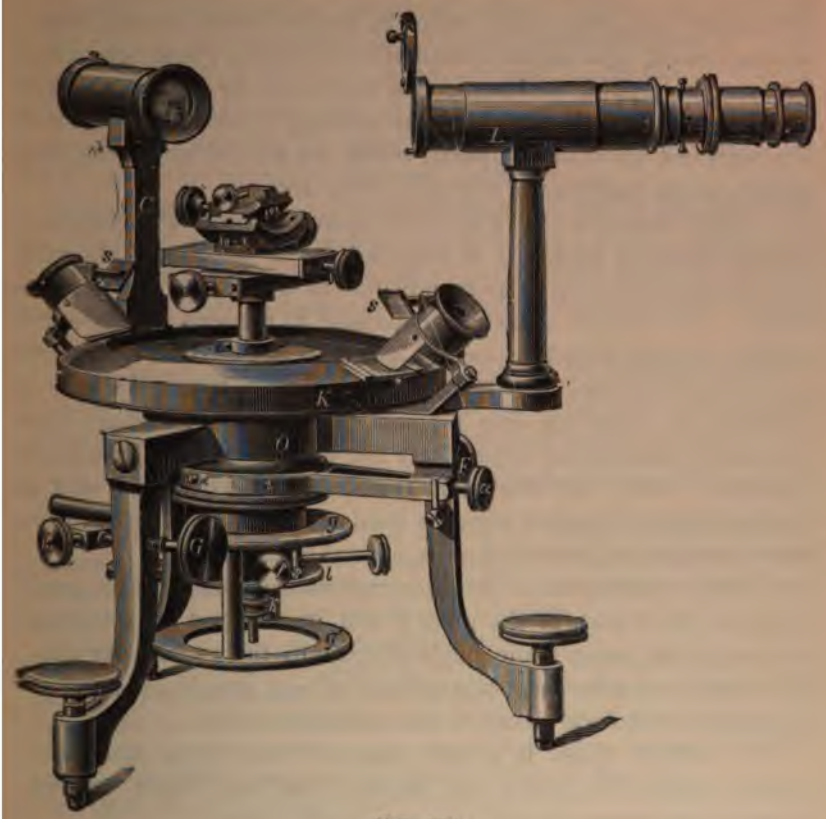


FIG. 193.

conveniently by g';* the fine adjustment is by the tangent screw, G, and the clamp screw, β,—*the inner axle* guiding the rod of the crystal carrier and which may be turned separately or clamped to the others.

The circle is graduated to half degrees. The vernier circle is protected by a ring, K, with glass windows over the verniers, while between the tripod and the vernier circle is a ring with two arms each with a magnifier and a mirror, s, by means of which the vernier can be read to half minutes and estimated to quarter minutes.

a. THE CRYSTAL CARRIER is like that described in the Mallard-Wollaston, the crystal being attached to a plate† by wax, and this to

* The part g' is omitted in many instruments and the clamping of the inner axle is by a vertical screw.

† Two or three sizes are furnished. For very small crystals platinum wire may be used with a cement of gelatine and acetic acid. By bending the wire the different zones may be adjusted.

two *cylinder arcs*, which are moved by tangent screws around intersecting axes at right angles to each other and to the goniometer axis. The common point of intersection is within the crystal, so that their motions tip the crystal without moving it much out of center. The arcs rest upon *sliding parts*, which are moved by micrometer screws, in the direction of the axes of the arcs.

c. THE TELESCOPE L. The observation telescope turns with the vernier circle, to which it is attached by a pillar, and its axis always intersects and is perpendicular to the axis of the goniometer. The vertical cross hair is parallel to the axis. Before the infinite distance objective swings an extra lens of a focal length equal to the distance to the goniometer axis. When in use this converts the telescope into a weak microscope. There are four eye pieces magnifying respectively six diameters, three diameters (generally used), two diameters (for distorted reflections) and a diminishing combination which is used with very small faces to reduce the signal to one third of its natural size.

d. The COLLIMATOR. The infinite distance collimator is fixed firmly to the tripod by the pillar *C* and its axis and the telescope axis are in the same plane perpendicular to the goniometer axis. The signals are either separate tubes or all four may be on a revolving target in the focal plane of the collimator lens.

The *Schrauf signal* is a light diagonal cross on a dark background, with, at the center, very minute cross hairs which are used in the adjustments and with rare very perfect faces.

The Websky Signal is an orifice bounded by two circular discs of equal diameter, the distance apart of which is regulated by a screw. This admits more light than a narrow cleft though still very narrow in the center and if dilated by a narrow face (see p. 23) and diminished in intensity the shape may still be divided symmetrically with some accuracy.

The Slit Signal is an adjustable vertical cleft especially adapted to broad faces or for measurement of indices of refraction.

The Pinhole Signal is a round ½mm. opening used in adjusting and in recognizing slight deviations from the zone or, by multiple reflection, cracked and facetted faces.

After the instrument is in adjustment* once for all the process of measurement is as follows:

*The complete adjustments are:

1. *Adjustment of the vertical hair.*

Several of the smallest and most brilliant crystals are selected, carefully cleaned with chamois skin and thereafter handled only with pincers or a tapering pencil of wax. Sketches are made showing striations, flaws and other peculiarities as revealed by the magnifier and later by the micro-telescope. Letters are assigned to all faces and thereafter represent them.

The most prominent zone is first measured and is designated by the letters of its two most prominent faces, say [*A C*]. The crystal is fastened with a conical pencil of wax to the plate so that the zone axis is approximately vertical and one face is parallel to one of the motions of translation.

The crystal is brought into view by the extra lens and raised or lowered by the mother screw *k*. With one arc at right angles to the telescope axis one of the edges of the zone is made coincident with

---

A needle is adjusted in the axis of the goniometer, so that during rotation it does not appear to move.

An eyepiece is adjusted so that its cross hairs are distinct and is then placed in the telescope tube, focussed on some distant object, the extra lens dropped into position and the eyepiece turned until one hair is parallel to the needle, then the cross hairs are moved horizontally until this hair coincides with the needle.

Making Telescope axis normal to Goniometer axis.

A plate of parallel glass is fastened to the holder approximately parallel to the goniometer axis and perpendicular to the telescope axis and is made exactly so by reflecting light upon the cross hairs from another plate fastened in front of the eyepiece at about 45° to the telescope axis. The reflected light throws an image of the cross hairs upon the parallel glass, which is reflected back into the telescope.

By aid of the arcs the images reflected from opposite sides of the parallel glass are made to take the same position in the field, that is, the plate is made exactly parallel to the axis of the goniometer, and by raising or lowering the cross wires their center is brought into coincidence with its image as seen after reflection. The telescope axis is then normal to the plate and therefore to the axis of the goniometer.

Adjustment of Horizontal Hair.

During rotation of the outer axle the image of the intersection of the cross wires should move across the field in coincidence with the horizontal wire, if not, raise, lower, or turn the wire as needed.

The Collimator Adjustment.

The pin hole signal is first inserted, focussed, and its center made to coincide with the horizontal wire of the telescope by the screw which fastens the pillar *C* to the tripod. The vertical slit and Websky's signal are focussed and made symmetrical to the telescope cross hairs and finally the cross hairs of the Schrauf signal are brought into apparent coincidence by their traversing screws.

The Other Eyepieces.

Are adjusted by means of Schrauf's signal. All eyepieces and signals have sliding collars with a projecting tooth which fits into a notch in the tube of the telescope or collimator. The collar is tightened when the tube is in final adjustment and thereafter is fixed. (M. Websky, *Zeit. f. Krystallographie*, IV., 545.)

the vertical hair by means of the corresponding micrometer screws; then with the other arc at right angles to the telescope axis the same adjustment is made with the other screws, and this is repeated until during a rotation the edge and vertical hair appear to be one, and on raising the extra lens the image of the signal from any face in the zone is symmetrically bisected by the cross hairs.

Except in the case of very narrow faces* it is not necessary to recenter on each edge, but is sufficient by a slight motion with the centering screws to make the vertical hair coincident with an imaginary central axis within the zone.

With the telescope at some convenient angle to the collimator (100 to 120 degrees) and with only β unclamped, the graduated circle and crystal are turned together by the disc g until the reflected signal is seen through the telescope, then β is tightened, the signal moved by the tangent screw G until it is bisected by the vertical cross hair and the vernier is read and recorded.

The screw β is again loosened and the rotation continued until the signal is received from a second face, and this is centered by G and β and recorded as before and the difference between the two readings is the normal angle. The position of a third face is determined in the same way and so on around the zone.

At least three measurements of any angle should be made and preferably with different portions of the circle. For the second measurement when the signal from the second face is centered the clamp l is loosened and the crystal alone is turned until the signal from the first face is centered, then l is tightened and β loosened and the crystal and circle turned together as before. This is repeated before a third measurement.

It is important to assign a quality mark to each image of the signal. For instance, a distinct image may be 2, a poor image 1, a band or shimmer 0, and the proportionate value of any measurement may then be taken as the sum of the quality marks of the readings. These may be recorded in tabular shape, as in the following example:

* See page 23.

CRYSTAL 1. Zone [m m''']

Face.	Quality.	Readings. First. °	′	Second. °	′	Third. °	′	Mean Normal Angle.	°	′	Times to be counted.
m	1	137	24½	54	53½	332	21	*mm'*	82	31⅓	1 + 2 — 3
m'	2	54	53	332	21½	249	50	*m'a'*	48	45⅔	2 + 2 — 4
a'	2	6	6½	283	36	201	5	*a'm''*	48	44⅓	2 + 1 — 3
m''	1	317	23½	234	51	152	20½	*m''m'''*	82	28⅔	1 + 2 — 3
m'''	2	334	56	152	22	69	50½	*m'''a*	48	44⅓	0 + 2 — 2
a	0	867	11	103	38½	21	6	*am*	48	45⅔	0 + 1 — 1
m	1	112	24	54	53½	332	21				

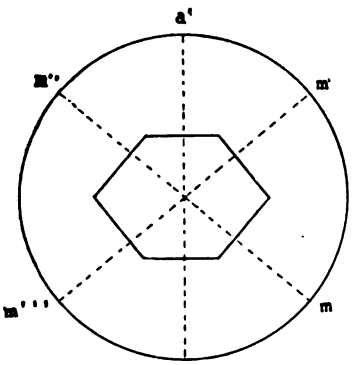


FIG. 194.

The angles indicate that *m*, *m'*, *m''*, *m'''* are faces of one form and that *a*, *a'* are faces of another form truncating the first as in Fig. 194. The most probable angles will be determined by averaging. For instance, the probable angle *m*–*m'''* may be the average of:

	° ′	Times to be counted.	Minutes.
* 2 *m a*	= 97 30⅓	½ or 1	30⅓
2 *m''' a*	= 97 28⅔	1 " 2	57⅓
2 *m' a'*	= 97 31⅓	2 " 4	125⅓
2 *m''' a'*	= 97 28⅓	1½ " 3	85
180 — *m m'*	= 97 28½	3 " 6	171
180 — *m''m'''*	= 97 31½	3 " 6	187
m m'''	= 97 29½	3 " 6	177
m' m''	= 97 29⅔	3 " 6	179
		34	1012

$$m - m''' = 97° 29\tfrac{7}{17}'$$

ERRORS DUE TO IMPERFECT CENTERING.

Let *E* denote the error.

Let *a* denote the angle between the crystal edge and the goniometer axis.

Let γ denote the angle obtained.

* The errors in the first four are already doubled, hence they can be allotted only half the sum of their quality marks.

Then will $E = \frac{a^2}{2\sqrt{2}} [\cos 45° - \cos (2\gamma + 45°)]$.

This value is a maximum when $\cos (2\gamma + 45°)$ is -1, that is, when $\gamma = 67\frac{1}{2}°$ for which $E = \frac{6}{10} a^2$ (approximately) and a minimum when $\cos (2\gamma + 45°) = +1$ or $\gamma = 157\frac{1}{2}$ for which $E = \frac{1}{10} a^2$.

This error is guarded against by the coincidence of signals from different faces of the zone.

In addition.

Let i denote angle of incidence of the light.

Let d and d' denote the distances of the faces from the axis.

Let r denote the distance to the light source.

Then will $E = \frac{d - d'}{r} \sin i$.

This diminishes as r increases or i decreases and is zero if r is infinite or $d = d'$; that is, the second error is eliminated by an infinite distance collimator, and even without it is only about $9''$ if $d - d' = \frac{1}{2}$ mm. and $r = 10$ m.

SPECIAL CASES IN MEASUREMENT.

Narrow faces tend to broaden the image by a phenomenon analogous to diffraction. The diminishing eyepiece may obviate this, but as the error is proportionate‖ to the eccentricity of the face, in such a case each edge must be separately centered.

Transparent Crystals give colored images due to total reflections within the crystal, if the crystal and telescope are turned steadily in one direction these images at a point move backwards (angle of least deviation); when such an image coincides with the true one it is only necessary to alter the angle of incidence.

Finely Striated Faces yield a bright colorless image with colored images on each side. The bright image is due to the plane tangent at the edges of the striæ and only when an image is obtained after the plane has been turned into some other zone is it proved that there is a definite crystal face underneath the striæ.

Bent or Cracked Faces yield distorted or manifold images. An risblende eyepiece will limit the reflection to a selected best portion, or an approximate measurement may be obtained from the outer member of each group, or by measuring in several zones

† Story Maskelyne's *Crystallography*, p. 414.

‡ Ibid, 402.

§ For other reasons i is commonly 50° to 60°.

‖ *Bull. Soc. Min. de France*, I., 35.

to the opposite face the properly oriented portion may be found. The best way is to use another crystal.

Twin Crystals and Composite Crystals in approximately parallel position will give good images in part of a zone and in the rest the images will be a little out of center.

*Dull Faces** may be coated with thin varnish or cover glasses may be glued to them or the extra lens may be used, the collimator slit narrowed to the smallest width which will give an illumination and the position of BRIGHTEST ILLUMINATION of both faces recorded several times. Traube's attachment to the collimator of the Fuess goniometer† shown in Fig. 195 may be used. It is

FIG. 195.

practically a little extra dark room; the light passing through the collimator and the conical tube *d* to the crystal. The cylinder *e* shuts out much of the extraneous light giving a darker field in which the luminous signal is by contrast brighter. Still more light can be shut out by laying a card over the top of *e* or better by the cap *g*, which may be turned to diminish the window *f* to a mere slit.

Small and nearly parallel faces which yield a combined image may be distinguished by changing the angle of incidence.

Crystals which alter in the air may be protected during measurement by replacing the crystal plate by a hollow cylinder‡ *a*, Fig 196, supporting a hemisphere *s* and plate *t*, the whole covered by a little glass bottle with bottom pressed in and ground to fit air-tight on the oiled hemisphere. In the channeled bottom is placed sulphuric acid or chloride of lime for deliquescent crystals, and

*Groth describes an elaborate application gomometer "das Fühlhebel goniometer" for measuring dull crystals *Physikalische Krystal.*, III. Ed., pp. 604–608.

†*Neus Jahrb. f. Mineralogie*, 1894, Bd. II.

‡*Jahrb. d. geol. Reichsanstalt*, 1884, 329.

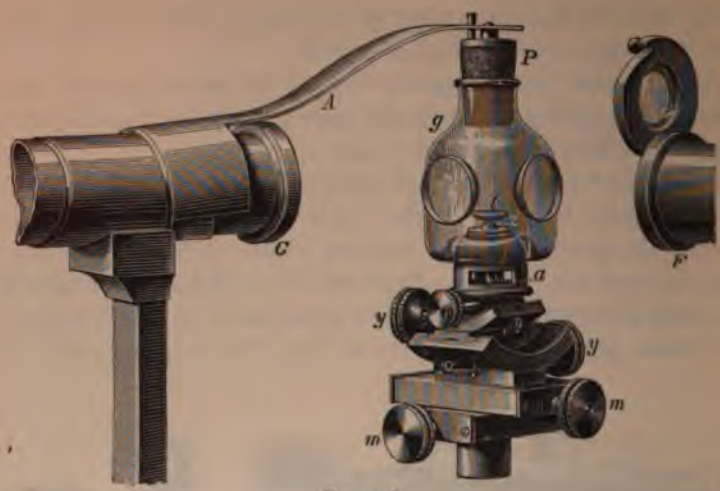


FIG. 196.

water, etc., for efflorecent crystals. The bottle is steadied by the spring clamp *A*, and the light enters and emerges normal to the plane windows of the bottle.

Imbedded crystals may be measured to within one degree by means of impressions in sealing wax.

THEODOLITE OR TWO-CIRCLE GONIOMETERS.

In the reflection goniometers described all faces of the zone which is placed perpendicular to the graduated circle reflect the signal and if the circle is considered to be the plane of projection their poles are in the circumference at angles apart equal to those measuring the rotation.

If a second direction of rotation at right angles to the other be added, *any* face in *any* zone may be referred to this first zone by the two motions necessary to bring it into the field, the one giving the point where its meridian cuts the primitive circle, the other giving the number of degrees on this meridian from the primitive circle. Czapski, Goldschmidt* and v. Federow have described such instruments; that of the latter, shown in Fig. 197, consists of a telescope *B*, which is also the collimator. The signals are on the revolving disc *b*, and the light entering at the focal plane of the objective passes through a small total reflecting prism and emerges as parallel rays. There is an extra lens *l* to bring the crystal into focus or, by

* *Zeit. f. Kryst.* XXI., 210–232.

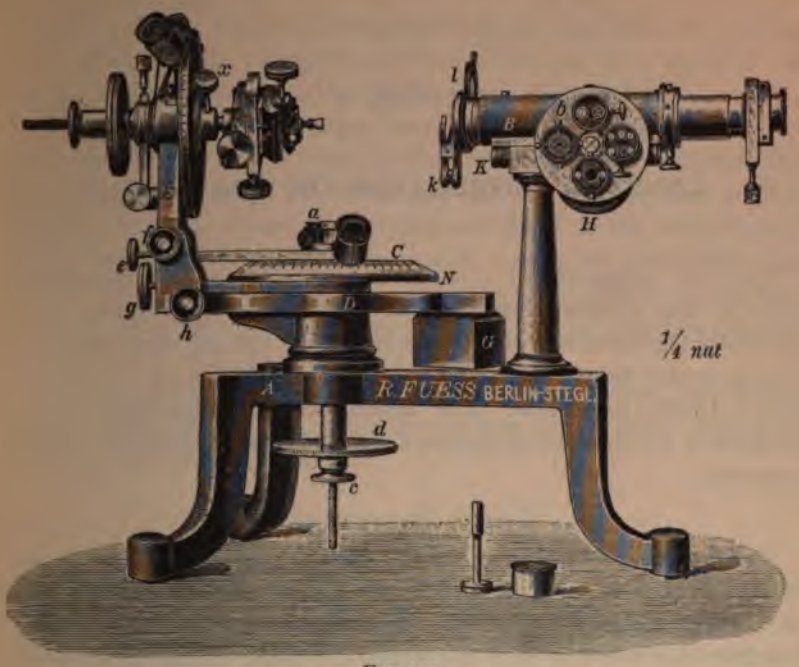


FIG. 197.

means of the spring clamp *h* and the rack and pinion *H* and *K*, one of the weaker objectives of a microscope may be focussed on the crystal. The eye-piece has an adjustable irisblende.

The stand consists of a tripod with a bored conical head in which rests the graduated circle *C* turned by *d* and read by the fixed vernier *N*, using clamp *e* and tangent screw *f*. The stand also supports *D*, the carrier of the vertical circle, which can be clamped to the stative by *g*, or to the horizontal circle by *e*.

The vertical circle is a complete Wollaston goniometer, but rotates with the horizontal circle when *e* is clamped. By transferring the centering apparatus to the horizontal circle the instrument may be used as a goniometer with vertical axis.

CHAPTER VI.

CRYSTAL PROJECTION OR DRAWING.

For the purposes of calculation and for a comprehensive view of the relations between the faces, the latter are more conveniently represented either by points, Fig. 198, as described pp. 20–24, under stereographic projections, or by lines as in the so-called Linear projection. For description and illustration some projection which actually figures the shape of the crystal is usually preferred.

Linear Projections.

In Quenstedt's linear projection* each face of the crystal is assumed to be moved parallel to itself until it cuts the vertical axis at a unit's distance above the the plane of the basal axes. The line in which the face then intersects the plane of the basal axes represents the plane.

For example, the crystal of topaz, Fig. 201, is orthorhombic in crystallization with $a : b : c = .529 : 1 : .477$. The basal axes are drawn at right angles, and the proportionate lengths .529 : 1 laid off upon them. As the crystal is symmetrical to the axial planes it is only necessary to determine the projections of the faces of one octant, say the upper right hand. The indices of these, therefore, are written, then their reciprocals, which are the axial intercepts (p. 12), and finally, these divided by the third term, that is, the intercepts of the faces when they cut the vertical axis at unity—

Indices.	Reciprocals.	Unit c intercepts.
111	111	111
110	11∞	001
120	$1\frac{1}{2}0$	001‡
223	$\frac{1}{2}\frac{1}{2}\frac{1}{3}$	$\frac{3}{2}\frac{3}{2}1$
041	$i\frac{1}{4}1$	$i\frac{1}{4}1$

* *Beiträgen zur rechnenden Krystallographie*, Tübingen, 1848.

† In Goldschmidt's Euthygraphic Projection the faces cut a unit's distance below the basal axes. *Uebrr Projection u. graph. Krystal.*, p. 25.

‡ Prisms are all projected as lines through the center *parallel to the projections of pyramids* of the same zone. In this case such a pyramid as 121.

The points on a and b corresponding to the first and second term of each unit c intercept, are then connected by a straight line which is the linear projection of the face. Fig. 200.

The projection of the edge between any two faces is evidently the line from the center of the projection to the intersection of the two lines representing the faces, and the direction of that edge is the line from the intersection to the unit point on the vertical axis. Hence all faces projected in lines with a common intersection intersect in parallel edges or lie in one zone. Evidently also a face lying in two zones will be projected in the line through the two common intersections or zonal points.

The zonal equations may be here used as described, pp. 17–20, and calculations may be made, though less conveniently than with the stereographic projection.

Projections in Parallel Perspective.

If the eye be conceived to be at an infinite distance the visual rays become parallel, and all parallel lines remain so * when projected, and the *proportionate* lengths into which any line is divided are not changed, although the line may be foreshortened.

Orthographic Parallel Perspective.

The plane of projection is at right angles to the visual rays. If the plane is the base the basal axes are full length, but if it is at right angles to the vertical axis the axis a in monoclinic crystals will be foreshortened to $a \sin \beta$ and in triclinic crystals a will become $a \sin \beta$ and b will become $b \sin \alpha$.

The faces of the zone normal to the plane of projection will be projected as lines bounding the figure at their true angles. The edges between other faces will be parallel both to the *edge projections* (see above) of the linear projection on the same plane or to the tangent to the primitive circle at the intersection with the zone circle of the two faces. For instance, in Fig. 199, the direction of the orthographic projection of any edge, say [041, 223], is both that of the line Oc, Fig. 201, and of the tangent at O, Fig. 198.

* Lines parallel in projection are not necessarily so in the crystal, for all lines in a plane through two rays are projected in the same line.

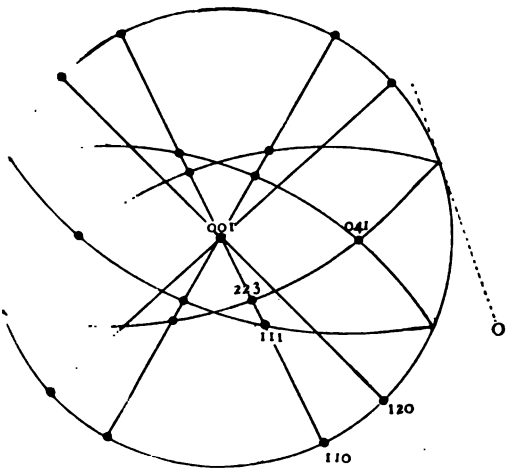


FIG. 198.

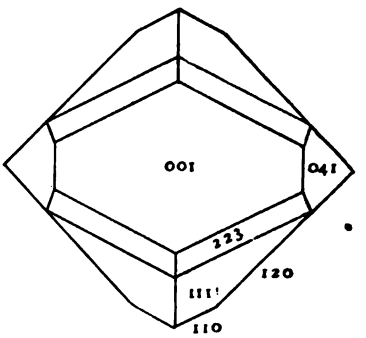


FIG. 199.

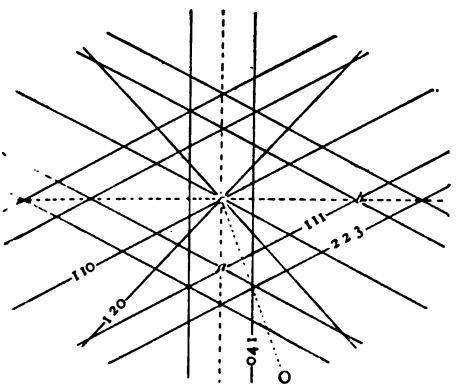


FIG. 200.

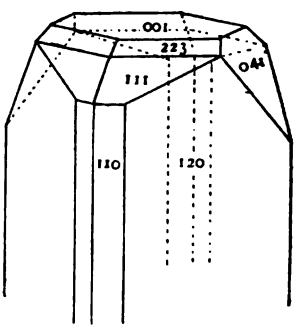


FIG. 201.

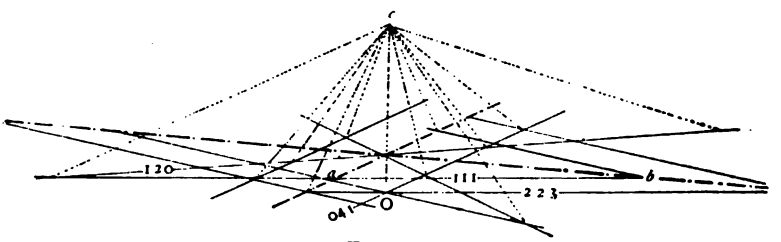


FIG. 202.

PROJECTIONS OF A TOPAZ CRYSTAL.

198. Stereographic.
199. Orthographic Parallel.
200. Linear.
201. Clinographic Parallel.
202. Linear upon axial cross.

Clinographic Parallel Perspective.

The plane of projection is oblique to the visual rays. This method gives an appearance of solidity and usually is made upon a vertical plane, the point of sight being to the right and above the crystal.

The Axial Cross, or perspective view of the axes, is first prepared.

If the angle to the right, or horizontal angle, is denoted by δ and the angle of elevation by ε, then the following relations* exist between these angles and the projected lengths and angles of the isometric axes in which the notation is as in Fig. 203.

$$OA = \sqrt{1 - \cos^2 \delta \cos^2 \varepsilon}\,; \quad OB = \sqrt{1 - \sin^2 \delta \cos^2 \varepsilon}\,; \quad OC = \cos \varepsilon$$

$$\cot AOC = -\delta \sin \varepsilon\,; \quad \cot BOC = -\tan \delta \sin \varepsilon\,; \quad \cot \delta = \sqrt{\frac{\cot AOC}{\cot BOC}}$$

$$\cos AOC = -\frac{\sqrt{(OA^2 + OB^2 - OC^2)\,(OB^2 + OC^2 - OA^2)}}{2\,OA.OC};$$

$$\cos BOC = -\frac{\sqrt{(OA^2 + OC^2 - OB^2)\,(OA^2 + OB^2 - OC^2)}}{2\,OB.OC};$$

The following table gives a series of projections, of which numbers 3, 4, 5, 6 and 10 are usually preferred.

Number.	Angles of Revolution.		Angles between projected Axes.		Foreshortened lengths of axes for true length = 1.			Approximate Proportionate lengths.		
	δ	ε	AOC	BOC	OA	OB	OC	OA	OB	OC
1	11° 35′	0° 49′	93° 58′	90° 10′	.200	.980	.999	10	49	50
2	14° 35′	1° 17′	94° 55′	90° 20′	.250	.968	.999	8	31	32
3	18° 26′	6° 20′	108° 19′	92° 06′	.239	.828	.994	29	100	120
4	18° 26′	7° 11′	110° 34′	92° 23′	.338	.950	.992	36	100	105
5	18° 26′	9° 28′	116° 17′	93° 8′	.353	.950	.986	37	100	104
6	19° 16′	2° 14′	96° 23′	90° 47′	.333	.944	.998	6	17	18
7	27° 59′	9° 50′	107° 49′	95° 11′	.493	.887	.985	5	9	10
8	45°	35° 16′	120°	120°	.817	.817	.817	1	1	1
9	20° 42′	19° 28′	131° 25′	97° 11′	.471	.943	.943	1	2	2
10	13° 38′	13° 15′	133° 24′	93° 11′	.324	.973	.973	1	3	3
11	10° 10′	10°	134° 6′	91° 47′	.246	.985	.985	1	4	4

* Story-Maskelyne's *Crystallography*, p. 480. Jos. Barrett, *Lehigh Quarterly*.

In drawing the cross, OC is vertical and the directions of OA and OB result from the angles AOC and BOC. The proportionate lengths may be laid off at any scale desired.

When cot δ and cot ε are simple numbers as in Nos. 3 and 5 in which cot $\delta = 3$ and cot $\varepsilon = 9$ and 6 respectively, the projection of the axial cross may be obtained as follows:

A horizontal line ss, Fig. 203, is bisected by a perpendicular, and perpendiculars are drawn as indicated at the ends and at points t and t' and distances laid off so that

$$Ot = Ot' = sg = \frac{Os}{\cot \delta} \text{ and } s'\varepsilon = Os \frac{\cot \delta}{\cot \varepsilon}$$

(in the figure cot $\delta = 3$, cot $\varepsilon = 6$).

The point e determines the line eO and AA'; the projection of the front horizontal axis is the portion of this line which is included between the perpendiculars at t and t'. The point g determines the radius Og, which is the length of half the vertical axis CC'. For the projection of the third axis draw Af parallel to ss', draw fO, and from the intersection v draw vB parallel to ss'. BB' completes the desired projection or axial cross.

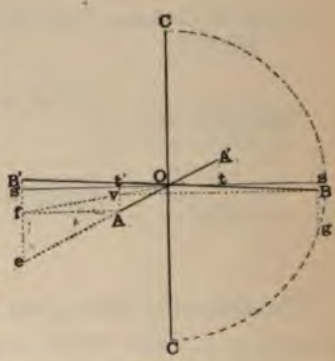


FIG. 203.

The axial cross for crystals of other systems is derived from the isometric cross by assuming the latter to be the projection of lines each equal in length to that axis of the crystal which extends conventionally from left to right (b).

In the TETRAGONAL and ORTHORHOMBIC crosses the proportionate values of the axes c and a are laid off on CC' and AA' respectively. In Zircon, for instance, the vertical axis is made $\frac{0.91}{0.64}$ of CC'.

In the MONOCLINIC crosses, Fig. 204, the vertical axis is a proportionate length on $C\overline{C}$, but $A\overline{A}$ is the projection of a line at right angles to $C\overline{C}$ and the *direction* of the clino axis is first found by laying off $Or = OC \cos \beta$ and $On = OA \sin$ and completing the parallelogram $OAtr$. The proportionate value of a is then laid off upon the diagonal Ot.

This and other projections are quickly obtained by using a metal quadrant.* Fig. 204 the center of which is at B. The edge AB and the arc AC are tapered to a thin edge for greater exactness in marking. With AB ten centimeters long the results will be correct within the limits of a drawing.

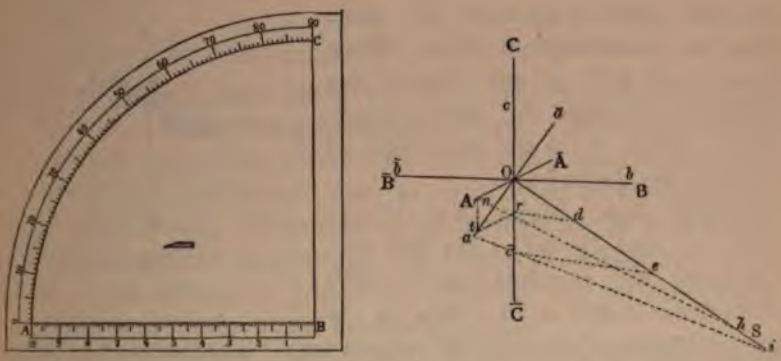


FIG. 204.

A scale line OS is drawn at will from the center O and axial lengths are transferred directly from AB to the scale line approximately to the third decimal. Sines and cosines are transferred as follows. If the edge BC and the scale line are made coincident and then, by use of a triangle, the quadrant is slid along in a direction at right angles thereto (BC remaining parallel to the scale line) until the scale line cuts the arc at the given degree and the approximate minute, the intercept on the scale line will be the sine. Similarly with the edge AB coincident with the scale line and a motion at right angles thereto, the intercept on the scale line will be the cosine.

All measurements are transferred to other lines, by lines parallel to the direction between the ends of the scale line and the unit line. For instance, in a monoclinic crystal in which $a:b:c=1.092:1:0.589$ and $\beta=74°\ 10'$. Oh is sine $74°\ 10'$ when radius is OS and transferred to OA is On; Od is cosine $74°\ 10'$ when radius is OS and transferred to OC is Or; by completing the parallelogram t and Ot result. For the axial lengths, Oi is 1.092 times OS and Oe is 0.589 times OS and transferred are respectively $Oa=1.092\times Ot$ and $Oc=0.589\times OC$.

* Described A. J. Moses, *Am. Journ. Science*, I., June, 1896.

In the *triclinic* crosses, Fig. 205, the same method is carried further, for instance: The constants for axinite are $a:b:c=0.492:1:0.479$, $a \wedge c=\beta=91° 52'$, $b \wedge c=a=82\ 54'$ (100) $\wedge$ (010)$=131° 39'$.

Vertical Axis. Make $Oc=OC\times .479$.

Macro Axis. Make $Oe=OB$ sin 131° 39′, and $Od=OA$ cos 131° 39′; complete the parallelogram *dOen*. Make $Oi=On$ sin 82° 54′ and $Ox=OC$ cos 82° 54′; complete the parallelogram *rOxb*. Then is *Ob* the projection of one-half the desired axis.

Fig. 205.

Brachy Axis. Make $Ol=OC$ cos 91° 52′, and $Op=OA$ sin 91° 52′. Complete the parallelogram; *pOlt* make $Oa=0.492=Ot$; then is *Oa* the projection of one-half the desired axis. These results are more quickly made with the quadrant previously described.

In the Hexagonal crosses, Fig. 206, the proportionate value of *c* is laid off on *CC′* and the basal axes are derived as follows:

Make $Op=OA=1.732$; draw *pB* and *pB′*; bisect *Op* by a line parallel to *BB′*; then are *OB*, *Oa* and Oa_3 the projections of desired semi-axes.

Fig. 206.

Determination of the Direction of Edges.

The unit form always results from joining the extremities of the axial cross by straight lines and other simple forms are easily drawn by methods which suggest themselves, for instance, the unit prism by lines through the terminations of the basal axes parallel to the vertical axis. It is always possible, also, to obtain two points of any edge by actually constructing the two planes and finding the intersection of their traces in two axial planes. The method, however, is cumbersome.

In all systems the projected intersection may be most easily found by the following method: A linear projection of the faces

is made (Fig. 202) upon the basal axes of the axial cross precisely as described for the ordinary linear projection. One point of the edge between any two planes is the unit point on the vertical axis; another is the intersection of the linear projections of the two planes and the line connecting these is the edge. For instance, *Oc*, Fig. 202, is the direction of the edge [041, 223] and is so drawn in Fig. 201.

CONSTRUCTION OF THE FIGURE.

The edges thus formed must be united in ideal symmetry, yet so as to show, as far as possible, the relative development of the forms.

A second axial cross is drawn parallel to that used in determining the edge directions and these are transferred by triangles, care being taken that all corresponding dimensions are in their proper proportions and in accord with the planes of symmetry. Generally it will be best to pencil in and verify the principal forms and later work in the minor modifying planes.

The back (or dotted) half of many crystals can be obtained by marking the angles of the front half on tracing paper, turning the paper in its own plane 180° and pricking through. This is also a test of accuracy, for the outer edge angle for angle should coincide.

TWIN CRYSTALS.

These have two set of axes, the second so related to the first that it corresponds to a revolution of 180° about the twin axis or line normal to the twining plane.

The two individuals may be in *apposition*, that is, the twin plane coinciding with the combination face. In this case the twin axis which passes through both centers normal to the twin plane will be bisected by the latter.

In interpenetrating twins the two centers may coincide or be near together. The orientation, however, will be as in the former case.

*Given the axial cross of one crystal to find that of a second crystal in apposition thereto.**

Let *OA OB OC*, Fig. 207, be the axial cross of the first crystal and *HKL* the twinning plane. To find first the point *Z* at which the normal from *O* cuts *HKL*, draw *H'L* and *HL'* parallel *AC* and draw *KL''* and *K'L* parallel *BC*. Complete the parallelograms

* Groth, *Physikalische Krystallographie*, p 599, III Ed.

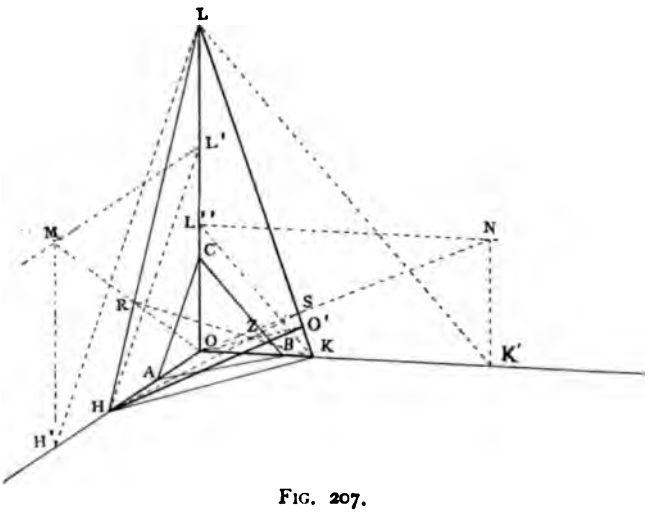


FIG. 207.

$OH'ML'$ and $OK'NL''$ and draw their diagonals OM and ON and from the intersections of these with HL and LK draw RK and SH respectively; their intersection is the desired point Z.

Because the crystals are in apposition prolong OZ till $ZO' = OZ$, then is O the center of axes for the second crystal and as the face HKL is common $O'H$, $O'K$ and $O'L$ are in direction and length the coordinates of this face on the new axes.

The unit lengths will be found by drawing AA', BB' and CC' parallel to $O'Z$.

All constructions on the axes of the second crystal follow the rules previously given.

PART II. THE OPTICAL CHARACTERS.

CHAPTER VII.

THE OPTICALLY ISOTROPIC CRYSTALS.

Light Rays.

In any homogeneous medium light is transmitted in straight lines, which are here called *light rays*, by vibrations of the particles of an imponderable light ether which fills all space even between the particles of solids, and it is probable that in common light the path of any vibrating ether particle is an ellipse constantly altering in shape and position but always remaining in the same plane.

The time of a complete vibration is called a PERIOD, the advance during a period is called a WAVE-LENGTH and determines the color of the light, and the AMPLITUDE of the vibration determines the intensity of the light.

Ray Surfaces.*

If we assume one vibration of a luminous point within a homogeneous material this motion will be transmitted along rays in all directions. At the end of any definite time along each ray the motion will have just reached a certain point. The closed surface through these points is called the ray surface of the substance. It is coincident with the wave surface but obtained in a different manner.

Ray Front and Front Normal.

The plane front of an extremely small pencil of rays including the given ray will be coincident with the plane tangent to the ray

* L. Fletcher, Optical Indicatrix, *Mineral Mag.*, IX., 291.

surface at the extremity of the ray and may be called the ray front. It is in general oblique to the ray. The direction normal to the ray front may be called the front normal.

Any homogeneous isometric crystal will show the same optical properties in all directions, that is, will be optically *isotropic*.

I. OPTICALLY ISOTROPIC CRYSTALS WHICH ARE SINGLY REFRACTING.

Ray Surface.

The velocity of transmission of light is constant in all directions, that is the ray surface is always a sphere, all diametral planes are planes of symmetry and all diameters are axes of isotropy.

In different substances, however, the velocities are different, that is for the unit of time with light of the same wave-length the spherical ray surfaces are of different diameters.

In the same crystal the shorter the wave-length of the light the slower the velocity of transmission; that is, for instance, for the unit of time the ray surface for violet light will be smaller than for red light.

Index of Refraction.

The differences in the ray surfaces are determined by measuring the amount a light ray is bent or refracted on entering a crystal; the ratio of the velocities of the incident and refracted rays or so-called "index of refraction" being equa to the ratio of the sines of the angles of incidence and refraction.

Let IO, Fig. 208, be a ray of light striking the crystal face AB at O. Let M_2 be the ray surface of the crystal, M_1 that of the outer medium described around the point of incidence for a unit

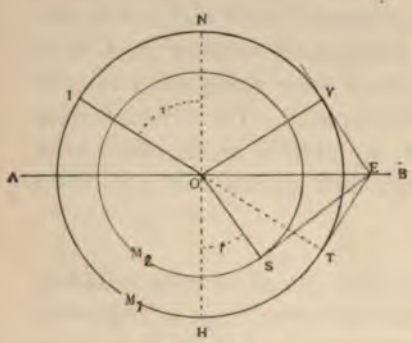


Fig. 208.

Fig. 209.

of time. The ray front in the first medium at the expiration of this time would be TE, E being a point of the front just impinging on AB, therefore also a point of the new ray front, that is of the tangent plane ES. The refracted ray is therefore * OS to the point of tangency. OY is the corresponding reflected ray.

In the triangles OET and OES, Fig. 208,

$$OE = \frac{OT}{\sin OET} = \frac{v_1}{\sin i} \text{ and } OE = \frac{OS}{\sin OES} = \frac{v_2}{\sin \rho}$$

hence,

$$\frac{v_1}{v_2} = \frac{\sin i}{\sin \rho}$$

Usually the ratio recorded is that of the crystal with respect to air. Denoting the velocity in air by v and the indices of the outer medium and crystal with respect to air by n_1 and n, we have

$$n_1 = \frac{v}{v_1} \text{ and } n = \frac{v}{v_2}$$

and substituting these values we have

$$n = n_1 \frac{\sin i}{\sin \rho}$$

There is no *refraction for normal incidence*, for $\sin i = 0$, hence

$$n = \frac{0}{\sin \rho}$$

which is only possible if $\rho = 0$.

With a *plane-parallel plate* the ray emerging is parallel to the entering ray, for at entrance, Fig. 210,

$$n = n_1 \frac{\sin i}{\sin \rho} \text{ and at emergence } n_1 = n \frac{\sin i_1}{\sin \rho_1}$$

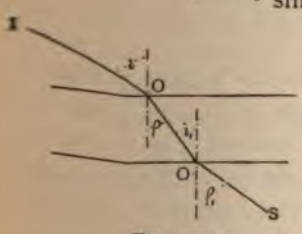


FIG. 210.

whence $$\frac{n}{n_1} = \frac{\sin i}{\sin \rho} = \frac{\sin \rho_1}{\sin i_1}$$

but $i_1 = \rho$ hence $i = \rho_1$

An approximate determination of the index of refraction may be made by measuring the displacement of the focal distance

* This is called the Huyghens construction, each point of the border surface becoming a new center of propagation of light. A still simpler construction is that of Snellius, Fig. 209, R_1 is a sphere with radius the index of refraction of outer medium, R_2, a sphere with index of crystal. Prolong IO to T. Draw TS parallel the normal ON, then is OS the refracted ray, and OY the corresponding reflected ray.

of a microscope caused by the interposition of a known thickness of crystal as described, p. 120, but usually one of the following methods will be employed.

Determination of Index of Refraction by Prism Method.

Let AOC, Fig. 211, be the section of the prism at right angles to the refracting edge O. About O describe the circles R_1 and R_2 with radii proportionate to the indices of refraction of the outer medium and prism respectively. Let IO be the incident ray, then, by the construction of Snellius (foot note p. 89), is OS the direction of the ray in the prism. From S draw SP normal to the surface CO, then is OP the direction of the ray on emergence. Denoting the prism angle by $\chi = NON^1$ the total deviation by $\delta = TOP$, the incident angle by $i = TON$, and the angle of refraction at the second surface by $\rho_1 = PON^1$.

$$\delta + \chi = TOP + NON^1 = 2\,TOP + PON + TON^1,$$
$$i + \rho_1 = TON + PON^1 = 2\,TOP + PON + TON^1$$

hence,

$$\delta + \chi = i + \rho_1 \text{ or } \delta = i + \rho_1 - \chi.$$

This value of δ is least * when $i = \rho_1$, then $\delta = 2i - \chi$ $i = \dfrac{\delta + \chi}{2}$.

When $i = \rho_1$ the ray RS, Fig. 212, within the prism must be normal to BD, the bisectrix of the refracting angle, therefore will $NRS = ABD$ or $\rho = \frac{1}{2}\chi$. Hence substituting in the formula for index of refraction $n = n_1 \dfrac{\sin i}{\sin \rho}$, we have,

$$n = n_1 \frac{\sin \frac{1}{2}(\delta + \chi)}{\sin \frac{1}{2}\chi}.$$

Practical Manipulation.—Two perfect faces of a clear transparent crystal are required, making such an angle (40° to 70°) with each other that *at the second surface* the ray is incident at less than the angle of total reflection, p. 90, or with a larger angle, the prism may be immersed in a strongly refracting liquid in a parallel walled

* The arc TP or δ, Fig. 211, cut by SP and ST, is least when they make equal angles with OS that is when the first deviation $i - \rho$ equals the second $\rho_1 - i_1$ for with any other position of IO the point T moves a certain number of degrees and one of the arms TS or PS approaches OS, becoming less oblique and cutting off a part TP, the other recedes becoming more oblique and adding a larger arc to TR, hence the combined change yielding a larger value for δ.

Glass vessel. If necessary, faces may be ground at the proper angles. The other faces of the crystal should be coated with lamp black.

FIG. 211.

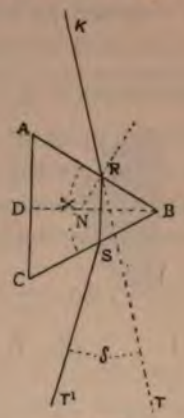


FIG. 212.

The most satisfactory instrument is a goniometer with vertical axis see p. 66–74. The angle χ is centred and measured as described, p. 69. The telescope is then clamped at T, Fig. 212, directly opposite the collimator K and a reading made, the crystal is moved by the centring screws so that the edge B is a little beyond the centre, and, the telescope and graduated circle remaining clamped, is revolved into, for example, the position shown in the figure. The telescope is then unclamped and turned towards the left until the image is in the field. The crystal is turned towards the right and if the image moves it is recentred by a movement of the telescope towards the right, and this double motion is continued until the image appears for a moment to be stationary and then starts to move in the opposite direction.

The position of rest T' is the position of least deviation and $TT' = \delta$

Monochromatic light* is essential.

* Certain solids vaporized in the flame of a Bunsen burner emit light essentially monochromatic. Three very commonly used are RED $\lambda = 0.000670$ Lithium sulphate, YELLOW $\lambda = 0.000589$ Sodium sulphate, Green $\lambda = 0.000535$ Thallium sulphate. Purer light may be obtained by using portion of a spectrum. A. E. Tutton describes an instrument for thus producing light of any desired wave length of greater brilliancy than that yielded by a colored flame. *Proc. Royal Soc.* 1894, v. 55, p. 111.

The production of monochromatic light by absorption of the other colors is not possible. Blue cobalt glass permits to pass only blue and extreme red. Certain solutions absorb certain rays; aniline blue absorbs yellow; permanganate of potash, green; sulphate of copper, red; chromate of potash, blue. The light passed through a series of these may become essentially monochromatic, *e. g.*, Sulphate of copper, aniline blue and chromate of potash, leave green.

If the instrument used does not permit of independent rotation of the crystal the position of minimum deviation and the reading T' are obtained by alternate movements of the telescope and divided circle and the reading of the collimator or T is obtained last.

Determination of Index of Refraction by Total Reflection.

When the index of refraction n_1 of the outer medium is greater than n of the crystal there is a so-called "critical" angle of incidence for which the angle of refraction is 90°; that is, the refracted ray travels along the border surface.

If $\rho = 90°$, $\sin \rho = 1$; hence, $n = n_1 \sin i$ or $\sin i = \frac{n}{n_1}$.

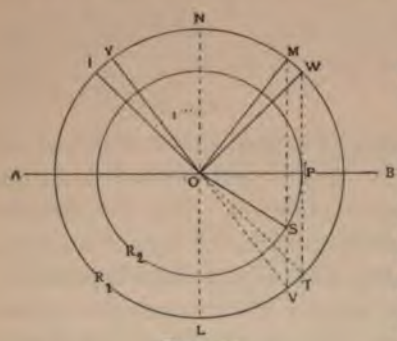


FIG. 213.

For any angle of incidence greater than this the light is totally reflected.

The following is the construction of Snellius: If AB, Fig. 213, is the border surface, R_1 and R_2 circles with radii proportionate to the indices of refraction of the first and second substances, then for the critical angle the direction of the refracted ray must be OP and from the tangent at P results the direction IOT of the limit incident ray.

According to the relative position of the observation telescope and the incident light two different results are obtained.

1°. TOTAL REFLECTION PROPER. If diffused light is admitted in the quadrant AN, Fig. 213, and the telescope axis is in the direction OW, all rays incident at less than the critical angle are in part reflected, *e. g.*, OY along OM and in part penetrate the crystal; while all rays incident at more than the critical angle are entirely reflected; that is, the telescope field receives on one half totally reflected rays, on the other partially reflected rays; between these is a sharp line, Fig. 214, which is the intersection of the focal plane of the telescope with a limit surface or cone the vertex of which is at O and the elements of which make the critical angle with the normal to the reflecting surface. This limit line is a curve, but

FIG. 214.

within the limits of the field of the telescope is essentially straight.

2. GRAZING INCIDENCE, or Observation of Transmitted Light.* If the light is shifted to the quadrant AL, or, which is equivalent, the light remains in AN and the telescope axis is made to coincide with OT and the transmitted rays are viewed, then, assuming the faces at entrance and emergence to be parallel, the incident and emerging rays are parallel. All rays incident at less than the critical angle are partially transmitted, but all of greater angle are totally reflected at the first surface; the telescope field is on that side dark, Fig. 215, but on the other side is illuminated by the rays incident at less than the critical angle.

FIG. 215.

In the Kohlrausch apparatus the substance is supported in a liquid of higher index of refraction than the crystal, in such a way that the reflecting surface is vertical and the crystals can be rotated about a vertical line in the reflecting surface. The crystal holder may be simply a metal plate with a window-like opening bisected by a platinum wire and adjusted once for all so that the back surface is in the desired position of the reflecting face and the wire coincides with the vertical cross-hair of the telescope. It is only necessary with this to fasten the crystal face over the window. A more elaborate holder permits rotation of the crystal in its own plane and other adjustments.

FIG. 216.

The rotation is observed by a horizontal telescope placed in the direction TO, Fig. 216, normal to a plane front of the vessel holding the liquid, and the rotation is recorded upon a graduated circle. The best position for the light is found by trial. When the sharp limit line between the totally

* Apparatus for observation of the transmitted ray with liquids were constructed by Christiansen, *Pogg. Ann.*, 1871, 143, p. 250 and others; and for solids by Quinke *Zeit. f. Kryst.*, 1879, 4, 540.

and partially reflected rays has been made to coincide with the vertical hair of the telescope the light and screen are moved to the opposite side and the plate is rotated until the limit line is again obtained and centered.

Since the angle between the telescope axis and the normal is the critical angle the rotation $NON^1 = 2i$ whence by $n = n_1 \sin i$ the index of the solid results.

A simpler apparatus is made as an attachment to the No. 2 Fuess Goniometer* and is shown in Figs. 217, 218. The pin *a* fits in place of the pin of the usual crystal plate of the instrument; provision for approximate adjustment is made but the accurate adjustments of the Fuess instrument are the principal reliance.

To secure approximately constant temperature the holder is covered by a box of asbestos with proper openings. The mineral is attached to the little plate *f*, and the control mineral at *g*. The rough adjustments are made by the eye, so that the necessary rotation can be secured; then the finer adjustment is made with the Fuess centring screws, and the vessel filled with the refracting liquid. The cover box is then put on, the light adjusted, and the boundary found and centred. After standing, say ½ hour, with light burning, the boundary is recentred and this repeated till no change takes place. The usual readings are then made, after which, without disturbing vessel or cover, the control mineral† is raised into the field by the vertical screw and readings obtained from it. From these the index results by the formula

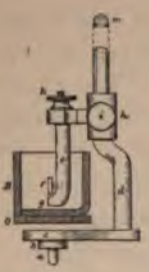


FIG. 217.

FIG. 218.

$$n = n' \frac{\sin i}{\sin I}$$

in which

i = half the angle of rotation of the mineral tested.
I = " " " " " " " control mineral.
n' = index of refraction of the control mineral.

* A. J. Moses and E. Weinschenk, *Zeit f. Krystallog.*, XXVI., 150 and *S of M. Quarterly*, XVIII., p 12.

† For a control mineral fluorite is very suitable, as its refraction has been carefully determined, and further, it is isotropic, has a low index of refraction and is easily obtained.

These instruments require either monochromatic light, or sunlight may be used if the eye-piece is replaced by a spectroscope, in which case at the proper angle the light of all colors will be totally reflected and the field will show on one side a bright spectrum, on the other a relatively dark spectrum, separated by a line oblique to the vertical hair, Fig. 219. The critical angles corresponding to the Frauenhöfer lines can be successively determined by bringing the points of intersection of these lines with the limit line into contact with the vertical hair.

FIG. 219.

The Liebisch apparatus, Fig. 220, employs a glass prism *P* of high index of refraction, firmly mounted* with the refracting edge vertical and one face normal to the axis of the holder. Diffused monochromatic light is admitted at the side *AB*, Fig. 221, reflected at the second side *BC* and emerges at the third side *AC*.

The edges of the prism are first made vertical; the crystal is then glued to the cap *z* and adjusted by the screws *q* so that a collimator signal from the crystal remains fixed in the telescope during a complete rotation by *T*. By the centring screws *y y* of the goniometer a central line of the crystal is made to coincide with the goniometer axis.

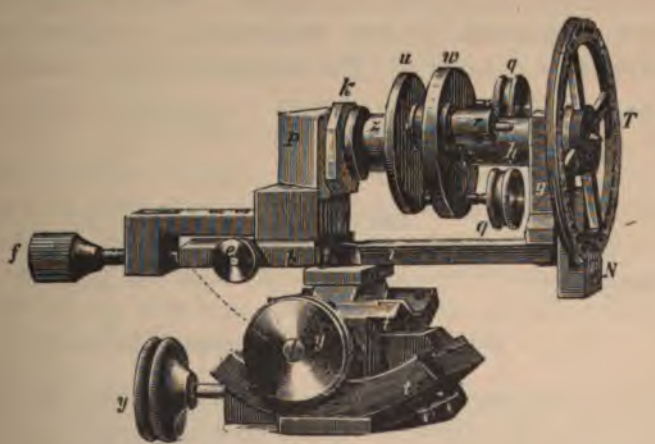


FIG. 220.

*As made by Fuess two prisms with indices 1.6497, 1.7849 are furnished.

The prism is then moved into contact with the crystal, close contact being secured by a drop of strongly refracting liquid.

The determination of the index of the plate requires the measurement of n_1 the index of the prism, of $\alpha = ACB$, the refracting angle of the prism, and of δ, the deviation of the emerging ray from the normal to the face of emergence.

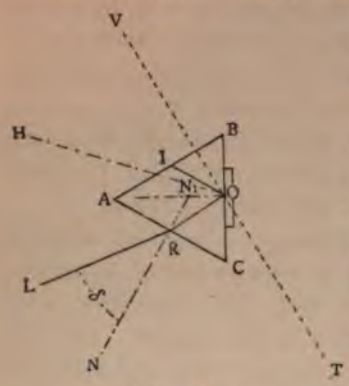


FIG. 221.

To determine δ three readings are needed: *First*, the telescope and collimator are placed opposite each other, say at T and V respectively. *Second*, the telescope is arbitrarily moved to some position L and the carrier turned until the limit line is centred. *Third*, the carrier is turned still further until AC gives a signal, that is, until RN bisects LV at OH.

This gives TL, NH, and $HL = \frac{1}{2}(180° - TL)$.

Then the deviation $\delta = LN = NH - HL$.

To find an expression for the index of refraction of the substance:

Since i is the critical angle of the prism, $n = n_1 \sin i$ (p. 90).

From the figure, $\alpha = ACB = AN_1R = N_1OR + N_1RO = i + \beta$, whence $i = \alpha - \beta$, or in general $i = \alpha \pm \beta$, since as n approaches n_1, β diminishes and may become negative; that is, the ray RL may be on the other side of RN. Substituting, $n = n_1 \sin(\alpha \pm \beta) = n_1(\sin\alpha\cos\beta \pm \cos\alpha\sin\beta.)$ The reflected ray OR is refracted at R

where $\dfrac{\sin\beta}{\sin\delta} = \dfrac{1}{n_1}$, whence

$$\sin\beta = \frac{\sin\delta}{n_1} \text{ and } \cos\beta = \frac{1}{n_1}\sqrt{n_1^2 - \sin^2\delta}.$$

Substituting,

$$n = \sin\alpha\sqrt{n_1^2 - \sin^2\delta} \pm \cos\alpha\sin\delta.$$

In the Pulfrich apparatus the glass prism is replaced by a vertical glass cylinder, the substance resting upon the upper base and illuminated from below by diffused light, no light being permitted to enter at the top. The angle δ of deviation from the normal on emergence is measured by a right-angled telescope revolving

on a horizontal axis. For this

$$n = \sqrt{n_1^2 - \sin^2\delta}.$$

In the Abbè apparatus the glass prism is replaced by a hemisphere of glass with the substance resting upon the horizontal base. The critical angle is measured directly by a telescope centred upon the centre of the sphere. For this $n = n_1 \sin i$.

The refracting liquid used may be: Thoulet solution, Rohrbach solution, α mono-Bromnapthalin, Methylene iodide alone or saturated with iodoform or sulphur or any other sufficiently stable and transparent liquid, the index of refraction of which is higher than that of the mineral to be tested.

	Density.	*Temp.*	*Li or B.*	*Na = D.*	*Tl or E.*	*Decrease for 1° increase.*
Glycerine.						
$C_3H_8O_3$	1.2594	20		1.47293		
Thoulet Solution.						
Potassium.	3.122	18	B 1.6960	1.7167	E 1.7391	
Mercury,	2.493	18	B 1.5855	1.6001	E 1.6160	
Iodide.	2.091	18	B 1.5129	1.5235	E 1.5347	
Rohrbach Solution.						
Barium.	3.564	23		1.7931	E 1.8265	
Mercury.						
Iodide.						
Bromnapthalin α.						
$C_{10}H_7Br$.		8		1.66264		.00045 Na
	1.4914	20		1.65820		
*Methylene Iodide.**						
CH_2I_2		8	Li 1.746	1.7466	Tl 1.7584	.00067 Li .00071 Na
		19		1.7421		.00073 Tl

II. OPTICALLY ISOTROPIC CRYSTALS WHICH ARE CIRCULARLY POLARIZING.

Certain isometric crystals possess the power of rotating the plane of polarization (see p. 100) of the incident light *whatever the direction of transmission;* they are therefore still isotropic, but doubly refracting, this rotation having been experimentally proved to be due to two rays transmitted with different velocities and circularly polarized in opposite directions.

RAY SURFACE.

Either circularly polarized ray is transmitted with a constant velocity in any direction, but with respect to each other the velocities have a constant difference, hence the ray surface must

* Saturation with iodoform raises the index about .02 and with sulphur as much as .04. Li line is at 32; B at 28. Tl line is at 68; E at 71.

be two concentric spheres. Since on reversing any section the direction of observed rotation is not changed it follows that at the extremities of any diameter of the ray surface the rotations in the same shell must be opposite in direction. There can, therefore, be no planes of general symmetry, though every diameter is an axis of isotropy.

The division is, therefore, necessarily limited to classes 28 and 29 which have no planes of symmetry. Examples in class 28 are barium nitrate, sodium chlorate and sodium bromate. The phenomenon has not yet been observed in class 29 and it is evident that symmetry is not the only determining cause.

The phenomena and testing of circularly polarizing crystals will be described more fully under uniaxial division IV.

ABSORPTION IN ISOTROPIC CRYSTALS.

In optically isotropic crystals monochromatic light diminishes steadily in intensity as the distance traversed increases, but is independent of the direction of transmission.

With white light the different component colors are absorbed at different rates.

A section of any given thickness therefore of an isometric crystal* will transmit the same color tint whatever the direction in which the crystal may be cut.

By decomposing this color with a prism the absorption spectrum is obtained, which usually shows a gradual change in absorption in adjoining portions, perhaps increasing from one end towards the other, perhaps increasing in both directions from the centre. With even a moderate thickness certain colors may be absorbed completely so that the spectrum shows dark bands.

* The color tints due to the combination of the partially and unequally absorbed rays may vary greatly in specimens of the same substance, which may be properly colorless or faintly colored in ordinary thicknesses and yet frequently occur of brilliant colors, which, nevertheless, conform perfectly in absorption to the crystal symmetry. It is probable that this is due to minute amounts of oxides of rarer metals titanium, zirconium, cerium, etc, dissolved like coloring matter in solution. See Weinschenk, *Zeit. f. Anorg. Chemie*, XII., 372.

CHAPTER VIII.

THE OPTICALLY UNIAXIAL CRYSTALS.

In every crystal of the hexagonal or tetragonal system the directions equally inclined to the crystallographic axis $\dot{c}$ are optically equivalent, so that $\dot{c}$ is an axis of isotropy and being a fixed crystallographic direction may be called an Optic Axis.* All diameters normal to $\dot{c}$ are axes of binary symmetry.

III. OPTICALLY UNIAXIAL CRYSTALS IN WHICH THE OPTIC AXIS IS A DIRECTION OF SINGLE REFRACTION.

Double Refraction.

In a moderately thick calcite† cleavage, Fig. 222, mounted with a rhombic face vertical and so that it can be revolved about a hor-

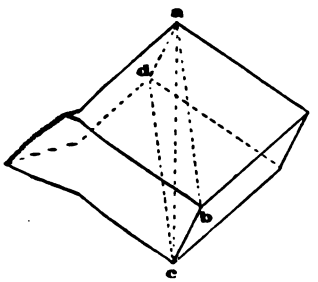


Fig. 222.

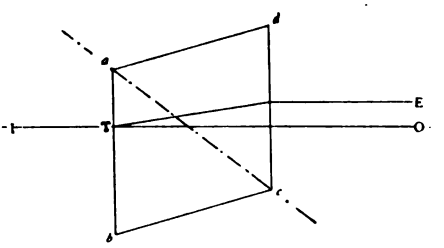


Fig. 223.

izontal axis normal to a vertical face, any light ray, *IT*, normally incident, Fig. 223, at the vertical face, is transmitted in the rhomb as two rays of essentially equal brightness‡ (giving two images of any signal), and as the rhomb is turned about the axis

*It will be seen later that while the optic axis in uniaxial crystals is fixed, the so-called optic axes in biaxial crystals change with the light or by heat or pressure.

†Calcite is chosen because of the marked divergence of the two rays. The discussions, however, are general.

‡Absorption is more marked in the case of one image than the other.

one of these remains fixed in position, the other moves around the first and always so that both remain in a plane parallel to *a b c d* (the so-called principal section) and at a constant distance apart.

With crystals optically isotropic and normal incidence the fixed image only would have been seen, hence this is called the *ordinary* and the other by contrast the *extraordinary*.

If a second calcite rhomb similarly mounted is placed in front of the first and revolved, the other remaining stationary, each of the two rays from the first is again split into two rays, an ordinary and an extraordinary, lying in the principal section of the second calcite, and these *are no longer of equal brightness*, but wax and wane in turn, the sum of their intensities remaining constant.

PLANE OF VIBRATION.

The changes in intensity (brightness) corresponding to different values of α, the angle between the principal sections, correspond exactly to the assumption that the varying elliptical vibrations of common light are converted by the first calcite into two sets of straight-lined vibrations, one parallel to the principal section, one at right angles thereto. Since the rays are of equal intensity, with equal vibration amplitudes, if we denote the ordinary and extraordinary rays from the first calcite by O and E and their ordinary and extraordinary components in the second calcite by O_0 O_ϵ and E_0 E_ϵ, it will be seen from Fig. 224.

FIG. 224.

	Ray.	Direction.	Amplitude.
If the principal section is the plane of vibration of the ORDINARY RAY.	O	ab	$Oa = 1$
	O_0	a_1b_1	$Oh = \cos\alpha$
	O_ϵ	c_1d_1	$Og = \sin\alpha$
	E	cd	$Oc = 1$
	E_0	a_1b_1	$Ok = \sin\alpha$
	E_ϵ	c_1d_1	$Of = \cos\alpha$
If the principal section is the plane of vibration of the EXTRAORDINARY RAY.	O	cd	$Oc = 1$
	O_0	c_1d_1	$Of = \cos\alpha$
	O_ϵ	a_1b_1	$Ok = \sin\alpha$
	E	ab	$Oa = 1$
	E_0	c_1d_1	$Og = \sin\alpha$
	E_ϵ	a_1b_1	$Oh = \cos\alpha$

Both assumptions give, therefore, the same amplitudes for the four rays, viz: $O_0 = \cos a$, $O_e = \sin a$, $E_0 = \sin a$, $E_e = \cos a$, therefore both correspond to the same relative intensites (proportionate to squares of amplitudes), moreover $O_0 = E_e$ and $O_e = E_0$ for all values of a, and also $O_0 + O_e = O$ and $E_0 + E_e = E$ since $\sin^2 a + \cos^2 a = 1$.

We shall hereafter assume* *that the plane of vibration of the extraordinary ray is parallel to the principal section, and that of the ordinary is at right angles to the principal section.*

FIG. 225.

The results corresponding to different values for a are illustrated in Fig. 225, $a\ b$ representing the principal section of the first calcite $a_1\ b_1$ that of the second calcite and the diameter in each circle being the assumed vibration direction. The intensities corresponding are proportionate to the squares of the vibration amplitudes.

Angle	Intensity proportionate to $\cos^2 a$		Intensity proportionate to $\sin^2 a$	
	O_0	E_e	O_e	E_0
0	1	1	0	0
45°	½	½	½	½
90°	0	0	1	1
135°	½	½	½	½
180°	1	1	0	0
225°	½	½	½	½
270°	0	0	1	1
315°	½	½	½	½
60°	¼	¼	¾	¾

That is at zero two rays are extinguished and the same two at 180°. From these points they gradually increase at the expense of the

* This supposition is purely for convenience as best connecting this work with the common usage in "Optical Mineralogy and Petrography." The question is one for the physicists and by them seems to be more generally decided in the other way. See Jas. MacCullagh, *Trans. Royal Irish Soc.*, XVIII., *XXI.*, W. H. C. Bartlett, *Amer. Jour. Science*, Nov., 1890. F. Neumann, *Vorlesungen über der festen Körper una des Lichtäthers.*

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other pair and are at a maximum at 90° and 270°, the others being then totally extinguished. At all diagonal positions there are visible four rays of equal intensity, and for all other positions as at 60° the intensity of two rays are greater than those of the other two.

PLANE OF POLARIZATION.

Common light reflected at a particular angle of incidence characteristic of the reflecting substance acquires the same peculiar characters as the rays produced by double refraction.

If normally incident at a rhombic face of a calcite rhomb an ordinary (unrefracted) image is obtained when the plane of reflection (through incident and reflected ray), is parallel to the principal section of the calcite, an extraordinary when these are at right angles to each other and, for all other angles, both ordinary and extraordinary of varying intensity, just as with calcite.

Malus* described the reflected ray as POLARIZED with reference to the plane of reflection and called the latter the PLANE OF POLARIZATION of the ray.

In the same sense the two rays produced from common light by double refraction in calcite are said to be POLARIZED.

The principal section of the analyzing calcite becomes the plane of reference. *Whatever it is parallel to when a ray undergoes ordinary refraction is the plane of polarization of that ray*; that is, the plane of polarization of the *ordinary ray* is the principal section and the plane of polarization of the *extraordinary ray* is at right angles to the principal section.

RAY SURFACE.

By measurement of the indices of refraction for different directions of transmission it is found:

1°. That the velocity of the ordinary ray is constant.

2°. That the velocity of the extraordinary ray in any section through $\dot{c}$ varies for different directions of transmission. For the direction parallel to $\dot{c}$ it is equal to that of the ordinary and differs most for the direction of right angles to $\dot{c}$ andf or any other direction, as discovered by Huyghens for calcite, the velocity is given by the corresponding radius-vector of an ellipse the axes of which are the least and greatest velocities.

The ray surface for light of any wave-length is therefore a double surface, the extraordinary shell being an ellipsoid formed by the

Mem. de la Soc. de Strasbourg, 1811, I., 284.

revolution of the ellipse, the axes of which are the least and greatest velocities, about one of its axes; and the ordinary shell a sphere with a diameter equal to the axis of revolution of the extraordinary. This surface is symmetrical to all planes through the optic axis and to the diametral plane at right angles thereto.

Denoting the indices of refraction of the fastest and slowest rays, that is of the two rays transmitted normal to the optic axis, by a and γ and their vibration directions by $\mathfrak{a}$ and $\mathfrak{c}$,* there will be two cases arbitrarily distinguished as POSITIVE in which $\mathfrak{c}$ is parallel to the crystallographic axis c and NEGATIVE in which $\mathfrak{a}$ is parallel to c. In the former case, therefore, the extraordinary ray with vibration direction parallel c will be the slower ray and the extraordinary shell will be wholly within the ordinary as in Fig. 226, while in the negative surface the extraordinary ray is the faster and the extraordinary shell will enclose the ordinary as in Fig. 227.

THE OPTICAL INDICATRIX.

All the relations between the optical characters of a crystal can be expressed by the geometrical characters of the extraordinary shell or its equivalent,† the ellipsoid of revolution, the axis of revolution of which is the index of refraction of the extraordinary ray transmitted normal to c and the equatorial diameter the index of the ordinary ray.

FIG. 226, +, $c = \mathfrak{c}$.

FIG. 227, —, $c = \mathfrak{a}$.

In Fig. 226 let the section of the extraordinary shell represent the corresponding section of the indicatrix, then let IO be any direction of transmission. Draw ID tangent at I, OE parallel to

* Often called axes of elasticity.

† In indicatrix axis in direction $\mathfrak{a} = a$, in shell $= \frac{1}{\gamma}$. In indicatrix axis in direction $\mathfrak{c} = \gamma$, in shell $= \frac{1}{a}$. But $a : \gamma = \frac{1}{\gamma} : \frac{1}{a}$.

ID, *ED* tangent at *E* and *EN* normal to *IO*, then are *OI* and *OE* conjugate radii and the parallelgram *OEDI* is of constant area $Oc \times O\mathfrak{a}$, hence, $Oc \times O\mathfrak{a} = OI \times EN$. Let *OA* denote the normal at *O* equal $O\mathfrak{a}$.

Describe the circumscribing circle or section of the ordinary shell, then are *OI* and *OH* the velocities of extraordinary and ordinary ray v_e and v_0.

From $Oc \times O\mathfrak{a} = OI \times EN$, have $OI = v_e = \dfrac{Oc \times O\mathfrak{a}}{EN}$

From $OH = Oc$ and $OA = O\mathfrak{a}$, $OH = v_0 = \dfrac{Oc \times O\mathfrak{a}}{OA}$

that is $v_e : v_0 = \dfrac{1}{EN} : \dfrac{1}{OA}$ or $EN : OA = \dfrac{1}{v_e} : \dfrac{1}{v_0}$

Moreover, *EN* in the principal section and *OA* normal to it are respectively the directions of vibration of rays to which they correspond.

That is: For any diameter of the ellipsoid considered as a direction of transmission there are two points of the surface, the normals from which are also normal to that diameter. *These normals are at once the directions of vibration and the reciprocals of the velocities of the rays transmitted in the direction of the diameter.*

Derivation of Positive Ray Surface.—Upon $\mathfrak{a}$ and $\mathfrak{c}$, Fig. 226, the directions of vibration of the fastest and slowest rays, make $O\mathfrak{a} = \alpha$ $Oc = \gamma$. Oc is the axis of rotation and the ellipsoid resulting is the indicatrix.

Section $\mathfrak{a}\mathfrak{c}$ *of Ray Surface.*—For the direction Oc the two normals are $O\mathfrak{a}$ and $O\mathfrak{a}$, for the direction $O\mathfrak{a}$ the two normals are $O\mathfrak{a}$ and Oc, for any other direction, *OI*, the two normals are $OA = O\mathfrak{a}$ and *EN*, in which *EN* varies between $O\mathfrak{a}$ and Oc according to the direction of transmission.

According to the rule then this section of one shell is a circle with constant radius $\dfrac{1}{O\mathfrak{a}} = \dfrac{1}{\alpha}$ and of the other shell is an ellipse with axis in direction $\mathfrak{a} = \dfrac{1}{Oc} = \dfrac{1}{\gamma}$ and in direction $\mathfrak{c} = \dfrac{1}{O\mathfrak{a}} = \dfrac{1}{\alpha}$ corresponding exactly to Fig. 226.

Section $\mathfrak{a}\mathfrak{a}$.—For every direction of transmission two normals exist, Oc and $O\mathfrak{a}$; that is, this section of the double surface is two concentric circles with radii $\dfrac{1}{Oc} = \dfrac{1}{\gamma}$ and $\dfrac{1}{O\mathfrak{a}} = \dfrac{1}{\alpha}$.

Determination of Optical Characters.

The determination of the optical characters of a uniaxial crystal consists essentially in the *determination of the axes of the Indicatrix;* that is, of the principal indices of refraction (indices of the two rays transmitted normal to the optic axis c). The determination may be direct or indirect.

Direct Determination of Principal Indices of Refraction.

(a) Prisms with refracting edge B, Fig. 212, parallel to the optic axis give for minimum deviation, p. 88, a direction of transmission RS normal to the optic axis.

(b) In prisms with refracting edge B, Fig. 212, perpendicular to the optic axis and faces AB and AC equally inclined thereto the optic axis is BD and the direction of transmission RS for minimum deviation is normal to it.

Because the horizontal section aa of the ray surface is two concentric circles, the formula p. 89 holds good for both rays

$$\alpha = n_1 \frac{\sin \tfrac{1}{2}(\delta + x)}{\sin \tfrac{1}{2} x} \qquad \gamma = n_1 \frac{\sin \tfrac{1}{2}(\delta' + x)}{\sin \tfrac{1}{2} x}$$

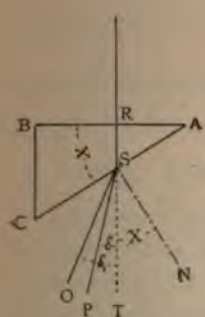


FIG. 228.

(c) In a prism ABC, Fig. 228, with one face AB parallel to the optic axis, rays normally incident* at that face experience no refraction, because in sections normal to the optic axis both shells are circles, but on emergence from the second face the two rays are differently refracted.

Denoting the prism angle by x, the deviation of the faster ray by δ and that of the slower ray by δ' and measuring these only we have:

$$\alpha = n_1 \frac{\sin PSN}{\sin TSN} = n_1 \frac{\sin (x + \delta)}{\sin x}$$

$$\gamma = n_1 \frac{\sin OSN}{\sin TSN} = n_1 \frac{\sin (x + \delta')}{\sin x}.$$

* With this face vertical and with collimator and telescope at any convenient angle obtain a signal from the face, then turn the crystal through one-half this angle until normal to the collimator.

(*d*) The indices α and γ may be calculated from the indices obtained with other prisms.*

(*e*) In *any* crystal face or section one of the extinction directions, p. 117, is in a plane through the optic axis and the other is at right angles thereto and, therefore, is itself at right angles to the optic axis.

If this direction is made horizontal in a total reflectometer the transmission will be at right angles to the optic axis and the methods and formulæ of pp. 90–95 will be available. There will be two distinct limit lines which may both be in the field at once as in Fig. 229 or may not, and are successively brought into coincidence with the vertical hair of the telescope.

FIG. 229.

The measurements determine the relative values of α and γ. By means of a nicols prism, p. 105, which transmits only light vibrating in a plane through its shorter diagonal, the ordinary and extraordinary rays may be distinguished, the former being transmitted when the shorter diagonal is at right angles to the optic axis, the latter when the shorter diagonal is parallel, as previously explained.

In positive crystals the ordinary is the faster, that is, corresponds to α.

In negative crystals the ordinary is the slower, that is, corresponds to γ.

Indirect Determination with Plane Polarized Light.

Parallel faced (plane-parallel) sections of known orientation are prepared. Cleavages are used when obtainable, or if the section is to be parallel to a crystal face this face is cemented to a glass and an opposite artificial face ground on with emery and polished with rouge. When the desired section is not parallel to any known face it is fastened to glass by slowly hardening cement, adjusted at the proper angle and ground, the new face being verified goniometrically with reference to other faces.

Crystals soluble in water are ground in some other liquid, as mono-bromnapthalin or benzine, and if fragile are ground only on a glass plate. After grinding the sections are cleaned and transferred to another plate.

A very perfect apparatus in which true planes may be rapidly

* Th. Liebisch, *Phys. Kryst.*, 1891, 384–390.

ground within 10′ of any desired direction has been described by A. E. Tutton.*

PLANE POLARIZED LIGHT may be produced from common light,

(*a*) By reflection at a particular angle of incidence (tan $i = n$), the vibrations being at right angles to the plane of reflection (plane through incident and reflected ray) in accordance with the assumption of p. 99.

(*b*) By refraction through a series of parallel glass plates, each plate increasing the proportion of polarized light. In this case the vibrations are in the plane of reflection.

In (*a*) and (*b*) common light is always present.

(*c*) By double refraction and total reflection of one of the rays. The best known device for securing this effect is the so-called Nicol's prism,† made from a cleavage of calcite with a length about twice its thickness, Fig. 230. The two small rhombic faces at 71° to the edge are ground away and replaced by faces at 68° to the edge. The prism is then cut through by a plane at right angles both to the new terminal faces and to the principal section. The parts are carefully polished and cemented by Canada balsam, the index of refraction of which is 1.54 or about that of the extraordinary ray *bd*, which, therefore, passes through the balsam with but little change in direction; the ordinary ray *bc*, however, with an index of refraction of 1.658, being incident at an angle greater than its critical angle, is totally reflected. The vibration direction of the emerging light is, therefore, parallel to the short diagonal of the face of the nicol, as shown by the arrow.

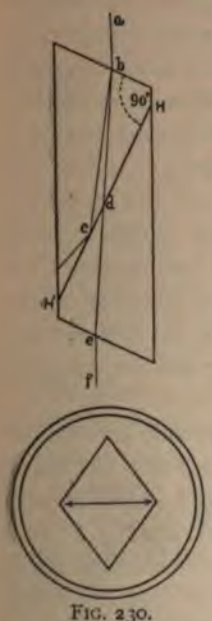


FIG. 230.

(*d*) By double refraction and absorption. Certain substances absorb one ray much more rapidly than the other, hence thicknesses can be

* *Proc. Royal Soc.* 1894. Vol. 55, p. 108.

† Described *Jamesons New Journal*, V. 6 1828. Various modifications of this prism have been made to decrease the cost and increase the field. See *Zeit. f. Kryst.*, XI., 179, 410, for instance.

The Foucault's prism uses a layer of air instead of Canada balsam and is cut at a different angle requiring a shorter prism but giving a smaller field.

In the Hartnack prism the terminal planes are at right angles to the axis, the prism is shorter and the field reaches 42°. It is much used.

The Bertrand prism is of flint glass with the high index of refraction of 1.658. It is bisected by a plane at 76° 43′ to the base and between the two halves is a thin calcite

chosen for which one is totally absorbed, the other is partially transmitted as light, the vibrations of which are in one plane. In tourmaline the ordinary ray is the more rapidly absorbed.

INTERFERENCE.—A ray of polarized monochromatic light *AB* Fig. 231, incident at the lower surface of a plane-parallel doubly refracting plate at any angle, is broken into two rays *BC* and *BD*, vibrating in planes at right angles to each other and following different paths in the plate. On emergence they follow parallel but not coincident paths and do not produce interference.

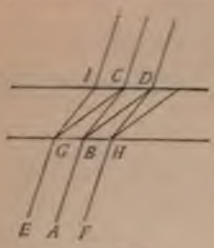


FIG. 231.

But among the other incident rays from the same source and parallel to *AB* there are rays *EG* and *FH*, such that from all points *C* and *D* of the upper surface there will emerge the ordinary component of one ray and the extraordinary of another following the same path. These rays will have travelled over slightly different paths in the plate with different velocities.

If a second polarizer is placed in the path of these rays each ray will be by it resolved into components the vibrations of which are in and at right angles to the plane of vibration of the polarizer and only the former will be transmitted. That is, there will emerge two rays advancing in the same line and with parallel vibrations. If these vibrations are alike in phase the intensity of the resultant ray will be proportionate to the square of the SUM of their amplitudes, but if unlike in phase the intensity will be proportionate to the square of their DIFFERENCE.

POLARISCOPES.—The instruments used for producing and studying the interference phenomena are called POLARISCOPES. In these parallel rays of plane polarized light, or converging bundles of parallel rays, are incident at one surface of the plate at a known angle, traverse the plate undergoing single or double refraction according to its nature; and, if doubly refracting, the rays following the same path are reduced by the analyzer to one plane of vibration producing interference phenomena.

The essentials of a polariscope for parallel light are shown in

cleavage properly oriented. The light enters the prism, and reaching the calcite is doubly refracted; the ordinary ray, with a refractive index about that of the glass, continues its course; the extraordinary with a much lower index is totally reflected. The field is about 45°. *Compte Rendu, Acad. Sci.*, Sept. 29, 1884.

Fig. 232. The mirror M sends parallel rays through the lower lens L, which concentrates them at the centre of the polarizer P; this point is also the focus of the equivalent upper lens L. On emergence from L the rays are again parallel, undergo refraction in the plate S of the substance and reach the analyzer A, which transmits only those components of the resultant rays the vibrations of which are in its own plane.

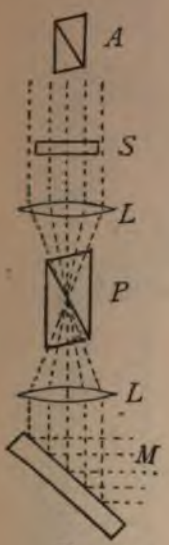


Fig. 232.

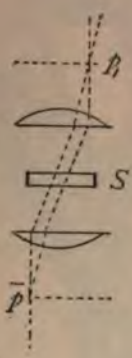


Fig. 233.

Convergent light is obtained by the addition of a lens or system of lenses of short focal length just above the plate S and a corresponding system just below the plate, Fig. 233. Any point p of the focal plane of the lower lens system is illuminated by a cone of rays the base of which is the lens. This cone is made a cylinder of parallel rays by the lens. The rays of each cylinder which traverse the plate are again concentrated by the upper lens system at points p', etc., are sorted by the analyzer and finally exhibit a picture or image the shape brightness and tints of which depend upon the structure of the plate for all the directions traversed by the cylinders of parallel rays.

The polariscope of to-day is usually a polarizing microscope. In the simpler types, such as the Seibert* 11 A, the polarizer below the stage can be raised, lowered and turned; the analyzer above the objective can be pushed in and out and convergent light images can be obtained with high power objective by placing a small convergent lens on top of the polarizer, raising the latter till it touches the section and removing the eye-piece. An orifice above the objective is always provided for the insertion of test plates, p. 146, and with increasing complexity there are added special micrometer eye-pieces, Bertrand lens for magnification of convergent light image, sliding motions of the stage and so on.

* For description of this instrument and its manipulation, see L. McI. Luquer, *S. of M. Quarterly*, 1896, p. 442–445.

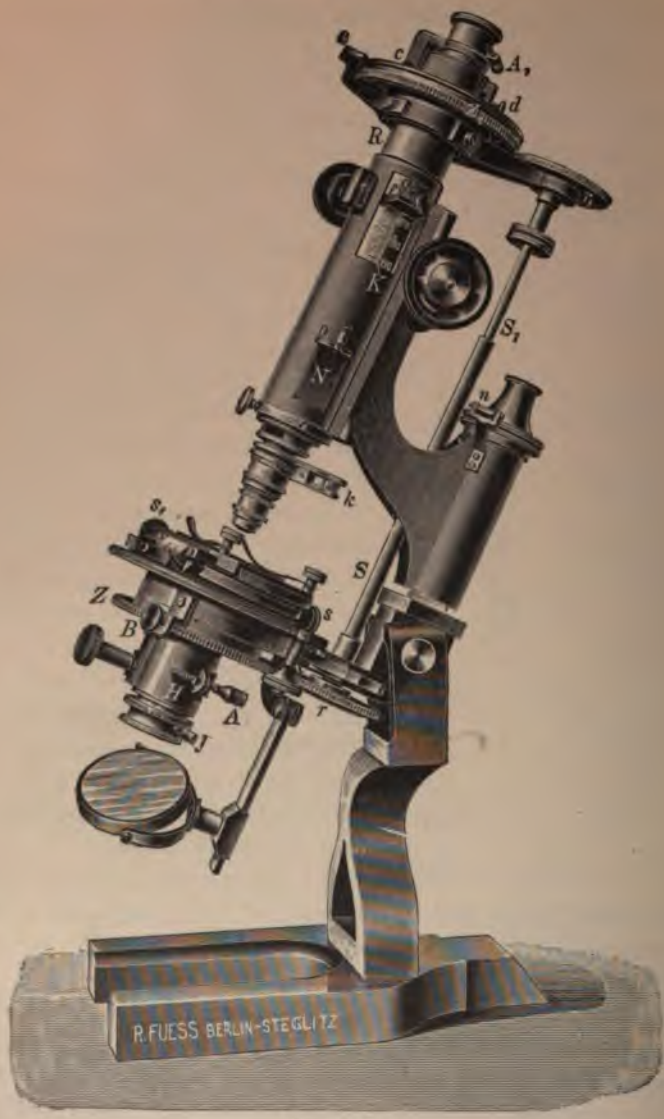


FIG. 234.

The Fuess microscope * model VI., Fig. 234, is at present probably the finest instrument made for this work. The stage reads to minutes and has quick rotation by hand, slow rotation by ratchet and sliding motions in two directions. There is an independent

* *Neues Jahrbuch f. Mineralogie* Beilage, Bd., X., 180, by C. Leiss.

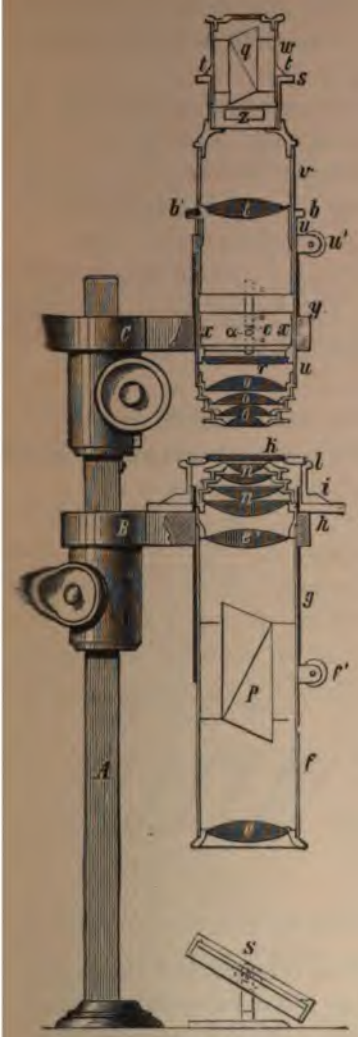


FIG. 235.

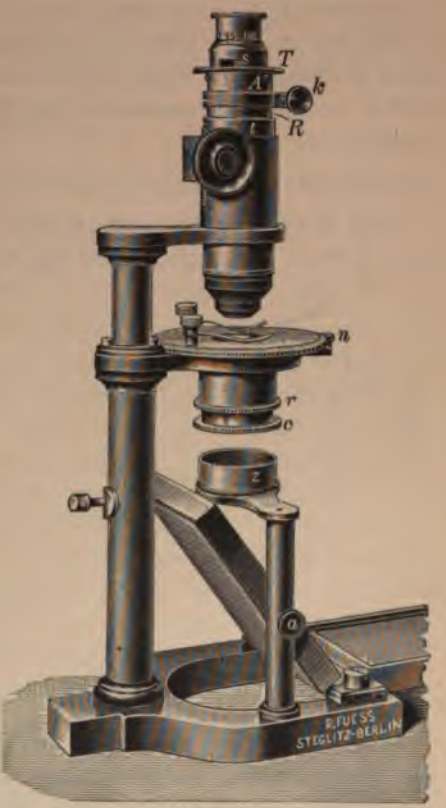


FIG. 236.

focussing screw for the interference figure and a special device of cog wheels $r\ Z$, $r^1\ Z^1$, by which there may be a simultaneous rotation of polarizer and a special cap analyzer, the object remaining at rest, but the same relative change taking place as if the stage were revolved and the nicols at rest.

Two forms of the Norremberg apparatus as constructed by Fuess are here shown. Fig. 235 shows the so-called Universal

Apparatus, *cc'* are collecting lenses on each side of the polarizer; above *c'* are four plano-convex lenses, *n*, forming the condenser and just over these the stage.

In a separate tube system above are the objective, composed of four similar plano-convex lenses *o*, and at their focal plane the glass plate *r*, on which a cross and a scale are marked; the image there formed is magnified by *t* and viewed through the analyzer *q*.

By removal of *n*, *o*, *r* and *t* the apparatus yields parallel rays.

Fig. 236 shows a later less expensive type, in which the lower nicol is replaced by a pair of mirrors. The high cost of iceland spar is the principal reason for the change and the results are very satisfactory.

WITH PARALLEL MONOCHROMATIC LIGHT, AND CROSSED NICOLS.

With crossed nicols none of the light from the polarizer can pass through the analyzer and the field must be dark.

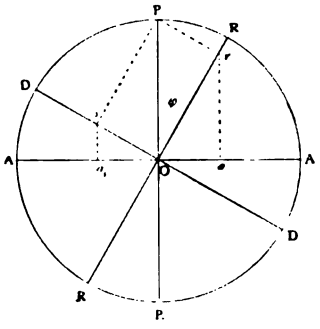


FIG. 237.

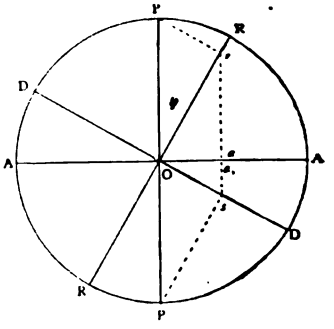


FIG. 238.

In sections normal to the optic axis the field remains dark throughout the entire rotation of the stage, and no interference phenomena are possible, because the light from the polarizer traverses the section in the direction of the optic axis, therefore, without change.

In all other sections there is double refraction and interference. The field is dark at intervals of 90°; that is, whenever the planes of vibration of the rays produced in the section coincide with the planes of vibration of the nicols. For all other positions the field is illuminated by the components of the rays which penetrate the analyzer and this brightening is most intense in the diagonal positions.

The rays pursuing the same path are by the analyzer brought into one plane of vibration and there interfere, the kind of interference being determined by Δ, the difference in the retardations which the two rays have undergone, the formula being *

$$\Delta = t\left[n_1 - n + \sin^2 i\left(\frac{1}{n} - \frac{1}{n_1}\right)\right]$$

Or for normal incidence,

$$\Delta = t(n_1 - n)$$

In which Δ is the retardation (difference in retardation) in $\mu\mu$ millionths of a millimeter.

t is the thickness of the plate in $\mu\mu$.

λ is the wave-length in $\mu\mu$.

i is the angle of incidence.

n_1 is the index of refraction of the slower ray.

n is the index of refraction of the faster ray.

When $\Delta = \lambda, 2\lambda, 3\lambda$, *etc., the field is dark during an entire revolution*, for Fig. 237, the components of PP, on emergence from the plate with vibration directions RR and DD, must be of the same phase that is the simultaneously displacing forces acting upon any ether particle O are Or and Os, which when reduced to the plane of the analyzer are Oa and Oa_1, in opposite directions and equal.

Fig. 239.

When $\Delta = \frac{1}{2}\lambda, \frac{3}{2}\lambda, \frac{5}{2}\lambda$, *etc., the light will be at its brightest* because the components of PP must then on emergence from the plate be of opposite phase, Fig. 238, and the simultaneous displacing forces acting on any ether particle O are Or and Os, which reduced by the analyzer to its plane are Oa and Oa_1 in the same direction and equal.

EXPERIMENT.

If a wedge of double refracting crystal, Fig. 239, cut so that its planes of vibration are parallel to the length and breadth, is placed between crossed nicols and illuminated by perpendicularly incident monochromatic light and there revolved.

It will be perfectly dark when in the normal positions and in all others will show a series of dark and light parallel stripes which are most marked in the diagonal position. If the nicols are made parallel the portions formerly light become dark. With light of a different wave-length, the distance between the dark bands is changed.

* Reduced from formula, p. 364, Glazebrook's *Physical Optics*.

*These relations may be deduced from the formula for intensity of emerging light

$$I = a^2 \sin^2 2\varphi . \sin^2 \left(\frac{\pi \Delta}{\lambda}\right)$$

in which

a = amplitude of incident ray, λ = wave-length, Δ = retardation, φ = angle between vibration plane of lower nicol and slower ray. I will be a minimum:

(a) when $\sin^2 2\varphi = 0$ or $2\varphi = 0°, 180°, 360°, 540°$, etc., or $\varphi = 0°, 90°, 180°, 270°$; that is, four times in a revolution of the plate or whenever the planes of vibration of the plate coincide with those of polarizer or analyzer.

(b) when $\sin^2 \left(\frac{\pi \Delta}{\lambda}\right) = 0$ which will be whenever $\frac{\Delta}{\lambda} = 1, 2, 3$, etc., that is whenever the phase difference Δ is a multiple of λ, for then $\sin^2 \left(\frac{\pi \Delta}{\lambda}\right)$ becomes $\sin^2 180°$, or a multiple thereof.

This is independent of φ, hence with this condition the plate will remain dark throughout an entire revolution.

I will be a maximum:

When $\sin^2 2\varphi = 1$ or $2\varphi = 90$, 270, etc., and $\varphi = 45°$, $135°$, $225°$, etc.

When $\frac{\Delta}{\lambda} = \frac{1}{2}, \frac{3}{2}, \frac{5}{2}$, for then $\sin^2 \left(\frac{\pi \Delta}{\lambda}\right) = \sin^2 90°, \sin^2 270°$, etc. $= 1$. This will be independent of φ and the illumination will exist throughout an entire revolution.

With Parallel White Light and Crossed Nicols.

In sections normal to the Optic Axis the field is dark throughout the rotation, the optic axis being the same for all colors.

In all other sections there is extinction every 90° and greatest brightness in the diagonal positions, but, since Δ may be at the same time approximately an even multiple of $\frac{1}{2}\lambda$ and an odd multiple of $\frac{1}{2}\lambda'$, light of one wave-length may be greatly weakened while that of another wave-length is practically undimmed; that is, there will result a tint due to unequal changes in all the colors.

As Δ increases in value it passes alternately through $\frac{1}{2}$, 1, 3/2, 2, 5/2 times each wave-length as shown in Fig. 240, in which the

*Th. Liebisch, *Grundriss der Phys. Kryst.*, 1896, p. 271–275.

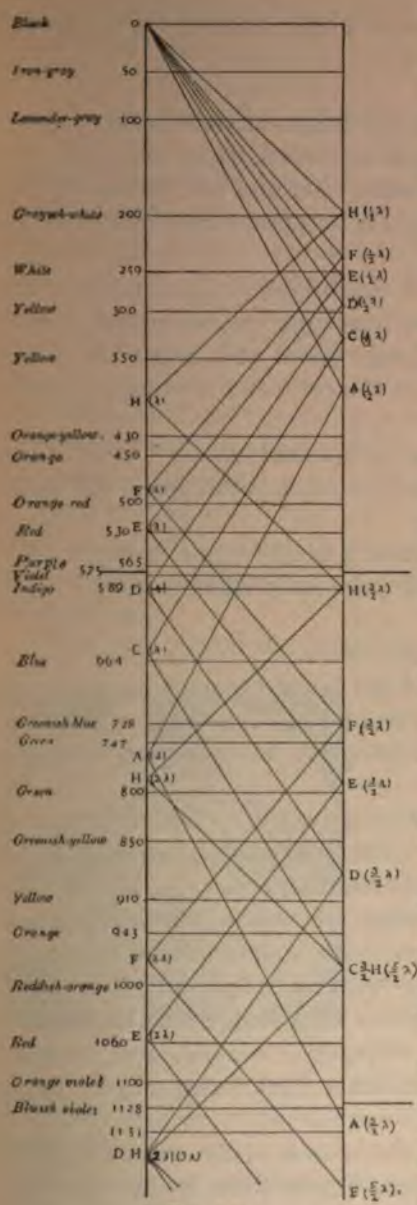


FIG. 240.

zigzag lines show the changes in brightness for six of the colors.* The tint corresponding to any value of J may be judged by noting which colors are near maximum and minimum.

FIRST ORDER COLORS.

$J = 0$.—All light is shut out; as J increases $\frac{1}{2}\lambda$ of violet is first reached and $\frac{1}{2}\lambda$ of blue and green next.

$J = 100\ \mu, \mu$.—The weakly coloring violet and still fainter blue and green give a total impression of *lavender gray*, which gradually brightens as the other colors show through.

$J = 259\ \mu, \mu$.—The color is *pure white*, after which the violet end begins to diminish in intensity and the red end to increase.

$J = 300\ \mu, \mu$.—Bright yellow is at a maximum and violet nearly extinguished. Green and red are weakened and together produce white, hence the predominating color is *yellow*.

$J = 450\ \mu, \mu$.—The yellow is still high, but the red rays are at or near their maximum and the other rays relatively weak; the color is therefore *orange*.

$J = 530\ \mu\mu$.—Violet and red are about equally near a maximum, green is extinguished and

* The values of λ for these in $\mu\mu$ (millionths of a millimeter) are H_1(violet), 393.3; F_1(blue), 486.0; E(green), 526.9; D_1(yellow), 589.5; C(red), 656.2; A(red) 760.4.

blue and yellow weak, hence the strongly coloring *red* predominates.

SECOND ORDER COLORS.

$\Delta = 575$ $\mu\mu$.—This is $3/2$ λ for the brightest violet, λ for brightest yellow and red and blue much weakened, hence the total impression is *violet*, often called *sensitive violet* and used in testing because with very slight change in Δ it becomes either red or blue.

$\Delta = 589$ yields *indigo blue*.

$\Delta = 664$ $\mu\mu$.—This is $3/2$ λ for the brightest blue and is λ for orange red and near it for yellow, hence predominating tint is *blue*.

$\Delta = 800$ $\mu\mu$.—This is $3/2$ for bright green and near λ for outermost red and 2 λ for violet, hence the color is a mixture of green, blue and yellow; that is, *green*.

$\Delta = 900$ $\mu\mu$.—This is $3/2$ λ for yellow and 2 λ for blue. Some red and violet emerge with the yellow; that is, the prevailing color is *orange*.

$\Delta = 1060$ $\mu\mu$.—This 2 λ for green, $3/2$ λ for some red, $5/2$ λ for indigo, and as red is the stronger color *red* predominates.

COLORS OF HIGHER ORDERS.

$\Delta = 1130$ $\mu\mu$ yields *sensitve violet No.* 2.

With increasing values for Δ the latter becomes an approximately perfect multiple of $1/2$ λ or λ for an increasing number of wave-lengths and the colors resulting are less pure and brilliant, for example:

$\Delta = 1590$ $\mu\mu$.—This is $5/2$ λ for orange red, $7/2$ λ for indigo, 3 λ for green, 2 λ for red, 4 λ for violet. The resultant total effect is a *red*.

With the still higher values this is further noticeable and beyond the fourth order the tints resulting are not to be distinguished from white. Hence thick crystals show no interference colors.

Experiment. The quartz wedge with white light will show colors in the order named, or the Federow mica wedge may be used, or assuming the value of $(n_1 - n)$ for special minerals and measuring t, the colors in plates of different thicknesses may be compared.

The colors of corresponding thicknesses of the mica wedge, in which $n_1 - n = .042$ $\mu\mu$ are much higher than for quartz with $n_1 - n = .009$ $\mu\mu$, one fifth the thickness producing the same retardation in mica.

The thicknesses corresponding to an interference color of red of first order, $\mu\mu = .009$, for several common minerals are in millimeters approximately.

Calcite	0.003	Quartz	0.060
Muscovite	0.013	Gypsum	0.061
Chrysolite	0.015	Orthoclase	0.079
Barite	0.048	Apatite	0.124

WITH CONVERGENT LIGHT AND CROSSED NICOLS.

The bundles of parallel rays, p. 107, each produce interference phenomena similar to those described, but since for oblique incidence

$$\triangle = t\left[n_1 - n + \sin^2 i\left(\frac{1}{n} - \frac{1}{n_1}\right)\right]$$

the values of Δ corresponding to odd or even multiples of $\frac{1}{2}\lambda$ will depend not only on the value $(n_1 - n)$ and the thickness, but upon the angle of incidence.

In sections normal to the Optic Axis there will be a dark cross, the arms of which intersect in the optic axis (centre of field) and remain parallel to the vibration planes of the nicols during rotation of the stage.

With monochromatic light, if the section is not too thin,* the optic axis will be surrounded by concentric circles alternately dark and light, Fig. 241. If the greatest value of Δ is less than λ no rings will show.

FIG. 241.

The distance apart of the dark rings decreases as the thickness of the section increases and as the distance from the centre increases, for both causes merging sooner or later into a uniform brightness.

With white light the rings become color rings strictly in the order of Newton's colors if the space permits, but often overlapping and finally merging into essentially uniform tints.

* In the accidental orientation of rock sections there is rarely found a perfect basal section and the thickness is frequently insufficient to produce rings. The uniaxial figure is nevertheless to be recognized in not too oblique sections by one or both arms of the straight armed cross which remain straight and parallel to the original position on rotation of stage, whereas in biaxial not only do the arms curve into hyperbola, but revolve in opposite directions to the rotation of stage.

The dark rings correspond to $\Delta = \lambda, 2\lambda, 3\lambda$, etc., and since (p. 100) the indices of the extraordinary ray are alike for all directions, equally inclined to the optic axis, $n_1 - n$, must be constant for any one value of i; that is, these rings must be circles.

The cross results from the planes of vibration of the different bundles being parallel and normal to different principal sections through bundle and optic axis. For any one position of the stage the vibration planes of certain bundles will be in the diagonal positions and those of others will coincide with the planes of the nicols; that is, the light circles will grade from greatest brightness in the diagonal positions to total darkness parallel to the nicols and as the stage is rotated successive rays will come into these positions, maintaining the same effect.

In Sections oblique to the Optic Axis the curves must be symmetrical to a principal section through the plate normal and the optic axis. If the optic axis shows in the field it will change its position with rotation of the stage, but the arms of the dark cross will always remain parallel to the planes of the nicols, Fig. 242. One arm only may show.

FIG. 242.

FIG. 243.

In sections parallel to the optic axis the curves are symmetrical to the principal section through the plate normal and optic axis and to a plane at right angles to the optic axis and are conjugate hyperbolæ, Fig. 243. They are usually only visible with monochromatic light.

WITH PARALLEL NICOLS.

The change from crossed to parallel nicols reverses all the phenomena of interference, p. 111. When $\Delta = \lambda, 2\lambda, 3\lambda$, etc., the

light is at its brightest and when $\Delta = \frac{1}{2}\lambda, \frac{3}{2}\lambda, \frac{5}{2}\lambda$, etc., the light is extinguished. That is with the quartz wedge the thicknesses which were dark bands in monochromatic light with crossed nicols will be light and the light bands dark and with white light the colors produced will be complementary to those obtained with crossed nicols.

With convergent light all bright portions of the field are extinguished and all dark portions become bright. With white light the complementary colors result.

If an interference color is passed through a prism the spectrum will show dark bands in the positions of the extinguished colors

Tests with Parallel Light and Crossed Nicols.

The polarizing microscope, p. 107, is nearly always used in determination, the cross hairs being made parallel to the previously determined vibration planes of the nicols.

DETERMINATION OF PLANES OF VIBRATION OR "EXTINCTION."

In any section of a uniaxial crystal the plane of vibration of the extraordinary ray passes through the optic axis and the corresponding extinction direction is the projection of that axis on the section; the plane of vibration of the ordinary ray is at right angles to this.

The positions of maximum darkness or extinction directions are always either parallel or symmetrical to cleavage cracks and crystal outlines.

A color contrast is more easily judged and may be obtained either by inserting a test plate of quartz or gypsum in the slot always provided between the nicols or by use of a special eye-piece such as the Bertrand.* The section is placed to cover only part of the field and for the extinction positions is of the same color as the rest of the field, but for any other position there is a difference of tint.

---

* Four quadrants which are equally thick basal sections of alternately right and left handed quartz; the lines of contact take the place of the cross hairs. Before the introduction of the crystal section the four quadrants are of the same color.

In the extinction positions the crystal section is colored like the rest of the field, but the slightest divergence from this raises the color of the section in two diagonally opposite quadrants and lowers the color in the other two. A special cap nicol must be used instead of ordinary analyzer.

Vibration Directions of Faster and Slower Rays.

With the extinction (vibration) directions in diagonal position, a test plate of some mineral, in which the vibration directions have been distinguished and marked, is inserted between the nicols (in a slot always provided) with these directions also diagonal. If the interference color is thereby made higher, the vibration directions of the corresponding rays are parallel; if the color is lowered, the corresponding directions of vibration are crossed.

Mica Test Plate or Quarter Undulation Mica Plate.—A thin sheet of mica on which is marked c, the vibration direction of the slower ray, which in mica is the line joining the optic axes. The thickness chosen is usually that corresponding to a blue gray interference color or say 140$\mu\mu$ which is $\frac{1}{4}\lambda$ for a medium yellow.

Gypsum Test Plate or Gypsum Red of First Order.—A thin cleavage of gypsum on which is usually marked a, the vibration direction of the faster ray. The thickness chosen corresponds to an interference color of red of first order or say 560$\mu\mu$, which is essentially λ for a medium yellow.

Quartz Wedge.—A thin wedge of quartz, cut so that one face is exactly parallel to the optic axis. The length of the wedge is parallel to the optic axis, and as quartz is positive this direction is c, the vibration direction of the slower ray.

The v-Federow Mica Wedge.—Fifteen quarter undulation mica plates superposed in equivalent position, but each about 2 mm. shorter than the one beneath it.

The mica plate raises or lowers the value of Δ by one quarter wave-length, the gypsum by about one wave-length and the two wedges by amounts increasing with the distance inserted.

When sections showing only low colors of first order are tested by the gypsum plate, there is practically considered the effect of the plate on the red of the gypsum.

Determination of the Retardation* Δ.

The v-Federow mica wedge inserted with corresponding vibrations directions crossed will for each interposed plate reduce Δ by 140 $\mu\mu$. To render the field dark will require $\frac{\Delta}{140} = n$ plates. Conversely $n.\ 140 = \Delta$, in which n is determined by count.

The quartz wedge similarly used will give an approximate value by counting the number of times the original color reappears, if n times, then is the color a red, blue, green, etc., of $n + 1$ order, for which the value may be looked up in a chart.

The Babinet Compensator consists of two equal quartz wedges

* Difference in Retardation.

A and B, Fig. 244, so cut that the optic axis c of say A is parallel to the length y and that of B is parallel to the breadth z. Any normally incident ray I will be divided in B into a faster (ordinary) ray vibrating parallel to y and slower (extraordinary) vibrating parallel to z. But reaching A the faster ray becomes the slower, and *vice versa*.

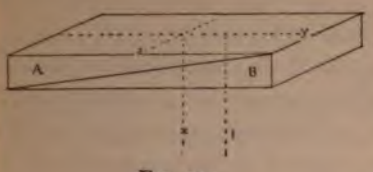


FIG. 244.

For the central position these will exactly balance, and with either monochromatic or white light there will be here a dark band with which a cross hair is made to coincide, and on each side, with monochromatic light, there will be at equal distances from this other parallel dark bands corresponding to $\Delta = \lambda$, 2λ, 3λ, etc.

Let A be made moveable by a micrometer screw and B be fixed and denote the movement necessary to bring the second dark band into coincidence with the cross-hair by δ, then δ corresponds to λ of the light used, and a movement of $n\delta$ corresponds to a difference of retardation of $n\lambda$.

With the compensator in the zero position and diagonal to the planes of the nicols, introduce a mineral section also in diagonal position and determine the motion D necessary to bring back the central band under the cross-hair, this change being due to the mineral, measures the value of Δ in the section, that is, $\Delta = \frac{D}{\delta}$ in wave-lengths, or $\frac{D}{\delta}\lambda$ in millionths of millimeter $\mu\mu$.

If on the scale used, δ is unity, then $D = \Delta$ in wave-lengths.

DETERMINATION OF THE STRENGTH OF THE DOUBLE REFRACTION.—If the thickness of the section is known $(n_1 - n)$ results from $\Delta = t(n_1 - n)$.

DETERMINATION OF THICKNESS OF SECTION.

From formula $\Delta = t(n_1 - n)$, t may be calculated if the value of $n_1 - n$ is known either for the crystal or for fragments of other known minerals ground with the section.

Moderately thick sections (0.5 mm. and upwards) may be measured by fastening on a thin cover glass larger than the section with thin balsam, cleaning away the balsam at the edges by alcohol and focusing successively on dust upon the lower surface of the cover and the upper surface of the glass. The difference in elevation is measured by the micrometer focussing screw and is t.

Measurements with a micrometer eye-piece, the section being set on edge are sometimes possible.

APPROXIMATE DETERMINATION OF PRINCIPAL INDICES.

The method of the Duc de Chaulnes* as improved by Sorby† depends upon the fact that the focal distance of a microscope is altered when a plane-parallel plate is inserted between the objective and the focus. Sorby focussed upon fine lines ruled on glass and placed some distance below the objective. Using the lower nicol only the indices corresponding to rays of definite vibration direction were determined either by measuring the displacement due to the unmounted section or successively those due to the glass alone and glass plus section. The thickness of the section and the displacement due to one revolution of the micrometer focussing screw must be known. Then denoting the thickness by t and the displacement by d

$$n = n^1 \frac{t}{t - d}$$

in which if the outer medium is air $n^1 = 1$.

Becke determines the relative indices of two substances in contact in a section by focussing upon the dark boundary line and raising the telescope tube, upon which the dark boundary appears to move towards the substance with the higher index of refraction.

TESTS WITH CONVERGENT LIGHT.

DETERMINATION OF CHARACTER‡ OF RAY SURFACE.

The MICA TEST PLATE inserted diagonally above a section normal

FIG. 245.

FIG. 246.

to the optic axis will destroy the black cross and break the rings

Mem. de l Acad. Paris, 1767–68. †*Mineral Mag.* I., 193, II. 1.

‡When the direction of the optic axis is known the character may be determined in *parallel light* by the extinction direction which is the projection of the optic axis. If this correspond to the slower ray, p. 118, the crystal is positive, if to the faster ray the crystal is negative.

into four quadrants, the relative effects in positive and negative crystals being shown in Figs. 245, 246. The corresponding signs + and — are suggested by the relative positions of the dark flecks and the arrow showing the direction ε of the test plate.

After insertion of the test plate a vertical plane through ε will contain the vibration directions of the slower ray in the mica, of the extraordinary rays in the quadrants passed through (first and third) and of the ordinary rays in the other quadrants (second and fourth).

In positive crystals the ordinary ray being the faster there will be an increase of $1/4\ \lambda$ in the first and third quadrants and a decrease of $1/4\ \lambda$ in the second and fourth.

At the centre Δ will now be $1/4\ \lambda$ and no longer dark. In the first and third quadrants the distances between the rings will be decreased about one-fourth by the increase of Δ by $1/4\ \lambda$. In the second and fourth quadrants by the lessening of Δ the spaces between the rings will be increased and near the centre the portion formerly bright with $\Delta = 1/4\ \lambda$ will become $\Delta = 0$; that is, two new dark flecks will be developed as in Fig. 245.

In negative crystals the extraordinary ray being the faster the phenomena are exactly reversed. The centre is light, the rings are narrowed in the second and fourth quadrants and widened in the first and third and in these two dark flecks are developed near the centre as in Fig. 246.

THE GYPSUM RED OF FIRST ORDER inserted with a parallel to the arrow is particularly advantageous for very thin or feebly refracting sections in which the rings are almost out of the field. The centre will be red. In positive crystals the first and third quadrants will lower the red to say yellow and in the seeond and fourth near the centre will raise the red to blue. In negative crystals the reverse will take place; that is, the "blue quadrants" correspond in position to the black flecks. This determination must be made in white light.

BY SUPERPOSITION OF A BASAL SECTION OF A MINERAL OF KNOWN SIGN. If the two sections are alike in character this will simply act like a thickening of the plate. If unlike they will partially neutralize each other. This test is little used.

CHAPTER IX.

THE OPTICALLY UNIAXIAL CRYSTALS (Continued).

IV. OPTICALLY UNIAXIAL CRYSTALS IN WHICH THE OPTIC AXIS IS A DIRECTION OF CIRCULAR POLARIZATION.

In certain hexagonal and tetragonal crystals monochromatic plane polarized light transmitted in the direction of the crystal axis *c* (optic axis) is not extinguished by *crossed* nicols, but is extinguished after a definite rotation of the analyzer.

PLANE, CIRCULAR AND ELLIPTICAL POLARIZATION.

If light with vibrations in one plane is decomposed into light with vibrations in two planes at right angles, any ether particle in the path of two interfering rays, p. 106, will be acted upon by two vibrations which will be at right angles, but, generally speaking, these will be neither of the same phase nor of the same intensity. The effects are analogous to those produced upon a swinging pendulum by a second impulse at right angles to the first.

1°. In general the resultant motion is elliptical and the light is *elliptically polarized*. For instance, the light emerging from a doubly refracting plate in a polariscope is in general elliptically polarized.

FIG. 247.

2°. If the phase difference is a half vibration ($\frac{1}{2}\lambda$), or an entire vibration, the resulting motion is in a straight line and the light is *plane polarized*. For example, Fig. 247, if AU is the original direction, OD and OC, the components at right angles $\frac{1}{2}\lambda$ apart, the resultant motion will be EI equal AU, but not in the same direction.

3°. If the vibrations are of equal amplitude, but possess a phase difference of $\frac{1}{4}\lambda$, that is, if one vibration is at the center of its swing when the other reverses its direction, the resultant motion is circular and the light is *circularly polarized*. For example, yellow rays emerge from a quarter undulation mica plate in the polariscope circularly polarized.

In the Fresnel rhomb, Fig. 248, the index of refraction of the glass and the angles of the rhomb are so related that a plane polarized ray S, normally incident, experiences a total reflection at E and another at U at such angles that at each reflection there is developed a phase difference of $\frac{1}{8}\lambda$* between the component vibrating in the plane of reflection and that vibrating in a plane at right angles thereto. That is, the vibration of any particle in the emerging ray T is the result of two component vibrations at right angles with a phase difference of $\frac{1}{4}\lambda$, but not in general of equal intensity. The ray is therefore *elliptically polarized* and if examined by a nicol will show variations in intensity, but never complete extinction.

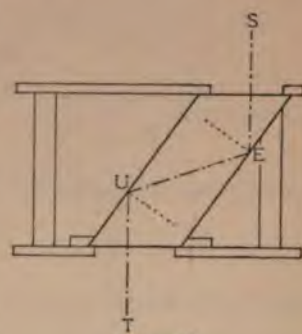


FIG. 248.

If the plane of vibration of the incident light is at 45° to the plane of reflection of the rhomb, the component vibrations will be equal in intensity, the emerging light will be *circularly polarized*, and the analyzing nicol will show no variation in intensity.

If two rhombs are used, the four successive total reflections develope a phase difference of $\frac{1}{2}\lambda$, that is, the impulses are in the same straight line, the emerging light is *plane polarized* and is extinguished whenever its plane of vibration coincides with that of the long diagonal of the nicol.

ROTATION OF THE PLANE OF POLARIZATION.

Two circular vibrations in opposite directions in the same plane are equivalent to a straight lined vibration of double amplitude and *vice versa*. A particle at A, Fig. 249, simultaneously subjected to two motions tending to carry it in any time t, at a uniform angular velocity through the equal arcs AR and AL would evidently in this time travel along the path $AU = 2AS$, for the components SL and SR at right angles to AS are equal and opposite and of no effect. For the arcs ART and ALT, the distance travelled would be $AN = 2AT$. Conversely, the vibration AN is equivalent to the two circular motions ART and ALT.

If a plane polarized ray be decomposed by any substance into two rays circularly polarized in opposite directions, these rays, in

* Due to difference in velocity and length of path.

traversing a plate of given thickness, will develope a phase difference, * and on emergence will combine to a plane polarized ray.

The planes of vibration (or of polarization, see p. 100) *of the incident and emerging rays will not be parallel.*

Let *AN*, Fig. 250, represent the vibration of the incident ray, *R* and *L* the advance points of the emerging rays at the moment

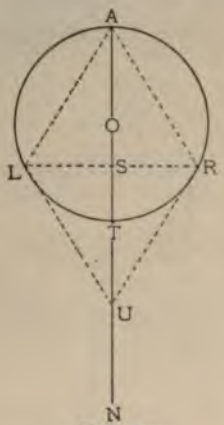


FIG. 249.

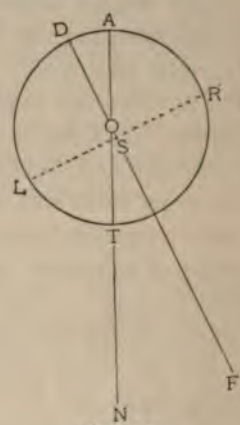


FIG. 250.

of emergence, then will *DF* be the vibration of the emerging ray, *D* being midway between *R* and *L*, for only for this direction will the components *SL* and *SR* neutralize each other.

The rotation for a thickness of 1 mm. is usually denoted by α and for any other thickness l, by $l\alpha$. The value of α in terms of the wave length and the indices of refraction of the two circularly polarized rays* is $\alpha=\frac{\pi}{\lambda}(n''-n')$.

THE OPTIC AXIS A DIRECTION OF DOUBLE REFRACTION.

In this division of uniaxial crystals, any incident plane-polarized ray transmitted in the direction of the optic axis, is converted into two diverging rays circularly polarized in the opposite sense (direction).

From the formula above given, the divergence $n''-n'$ may be calculated, or, even when minute as in quartz, it may be experimentally shown by the method of Fresnel with three prisms of right and left quartz cut as in Fig. 251.

In *A P D* the right-handed ray is the faster, reaching *P A C* it is the slower, hence is bent towards the normal while the left-handed

* Th. Liebisch. *Grundriss der Phys. Kryst.* 1896, p. 124.

ray is bent away from the normal. Reaching $A\,C\,E$ the right is again the faster and is bent from the normal while the left is bent towards it, and finally on emerging both are bent from the normal: all changes acting to increase the divergence until with $P\,A\,C = 152°$ a divergence of 4° is reached which can be measured by a goniometer.

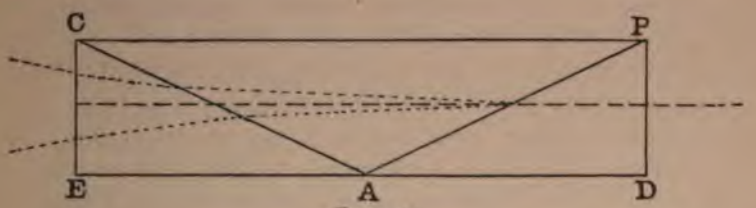


FIG. 251.

It can be shown by a nicol that these emerging rays are circularly polarized and in the opposite sense.

RAY SURFACE.

The ray surface is closely that of the preceding division, but with the ellipsoid and sphere not quite in contact at the optic axis, one of them a little flattened, the other a little drawn out.*

It is probable that there is a grading from plane polarized light transmitted normal to the optic axis, through elliptically polarized light, the eccentricity of the ellipse decreasing with the angle between the transmission direction and the optic axis, to circularly polarized light in the direction of the optic axis.

The optic axis remains an axis of isotropy, every diameter perpendicular to it is an axis of binary symmetry, but there are no planes of symmetry because the direction of rotation in corresponding shells at the extremities of the optic axis are reversed.†

The division is therefore limited to crystals in which the crystal faces are not symmetrical either to any plane or to the centre. A further condition seems to be that the right and left forms shall be "enantiomorphs" (the one can not by rotation about any axis be brought into coincidence with the other). For example, Group 17 has no planes of symmetry, but the forms are not enantiomorphs and circular polarization does not occur.

* Lang with quartz shows the velocity of the ordinary rays near the axis is not quite constant and that the extraordinary does not follow exactly the Huyghens law of common uniaxial crystals.

† The sense of rotation is not reversed on inverting the plate.

The classes in which circular polarization is possible are: TETRAGONAL SYSTEM. Class 9, to which no crystals have yet been referred. Class 10, circular polarization is not proved in any crystal of this class. Class 12, examples: strychnine sulphate, guanadin carbonate, diacetylphenol-thalein. HEXAGONAL: (RHOMBOHEDRAL DIVISION). Class 16, example: sodium periodate, $Na\ IO_4$. $3H_2O$. Class 18, examples: quartz, cinnabar, matico camphor $C_{12}H_{20}O$, tartrates of rubidium and caesium, subsulphates of lead, potassium, calcium and strontium; (HEXAGONAL DIVISION). Class 23, example: potassium-lithium sulphate, $K\ Li\ SO_4$. Class 24, circular polarization is not proved in any crystal of this class.

The direction of rotation corresponds to the right and left of the enantiomorphs.* With one or two exceptions, *e. g.*, sulphate of strychnine, circular polarization is destroyed by dissolving, fusing or vaporizing the crystal. Certain substances, however, showing in crystalline condition no power to rotate the plane of polarization show it in solution or in vapor, and are called optically active. These substances when in crystals are enantiomorphic right or left as the solution is right-handed or left-handed. If acids are optically active their salts usually are so.†

WITH PARALLEL MONOCHROMATIC LIGHT AND CROSSED NICOLS.

In sections normal to the Optic Axis the field will be illuminated, though only feebly in thin sections, and will only become dark when the analyzer has been turned an amount equal to the rotation

$$la = l\frac{\pi}{\lambda}(n'' - n').$$

That is, an amount increasing with the thickness and double refraction in the direction of the optic axis and decreasing with the wave-length. Approximately

$$a = \frac{A}{\lambda^2} + \frac{B}{\lambda^4} + \text{etc.}$$

* Herschel in *Trans. Cambridge Soc.* I, 43, showed that circular polarization in quartz was apparently produced by the same cause which determined the position of the so-called plagihedral faces, for a plate of quartz cut perpendicular to the vertical axis of a crystal in which the left plagihedral faces occur will turn the plane of polarization of the incident ray to the left and a similar plate from a crystal showing the right plagihedral faces will turn this plane to the right.

† See Pasteur's lectures before Société chimique de Paris 1860, published as No. 28, Ostwald's *Klassiker der Exacten Wissenschaften, Ueber die Asymmetrie.* Leipzig, 1891.

In all other sections there is extinction every 90° and brightest illumination in the diagonal positions. Extinction also takes place for $\frac{\Delta}{\lambda} = 1, 2, 3$, etc., as described p. 111.

With Parallel White Light and Crossed Nicols.

In sections normal to the optic axis, since the rotation increases as the wave-length decreases, the different colors will be dispersed and emerge vibrating in different planes. As the analyzer is turned its plane of vibration will be successively at right angles to the planes of vibration of the different colors. That is, at any time only one color can be shut out completely and the rest in varying degree. The color with vibrations most nearly parallel to the vibration planes of the analyzer will be least shut out and will determine the predominating tint in the color seen.

In all other sections there will be extinction every 90° and greatest brightness in diagonal positions and the color tint due to interference as described p. 113.

With Convergent Light and Crossed Nicols.

In sections normal to the optic axis, with monochromatic light, thin sections will show the dark cross the arms of which will always be parallel to the vibration planes of the nicols and somewhat indistinct near the centre. In thicker sections the bars will not reach the centre, Fig. 252, and no dark rings will form there, the first ring being some distance out, no matter how thick the section may be, beyond this there will appear the alternate dark and light rings with radii *nearly** as in the last division.

Fig. 252.

In general the inner circle is bright except when the thickness for the light used gives $l\alpha = 180° \times n$, in which $n = 1, 2, 3$, etc., when it is black. With white light the central circle has the color tint

* Exact measurements show that the radii of the circles near the centre do not obey the law of common uniaxial crystals. J. C. McConnell, *Phil. Trans. Roy. Soc.* 1886, 177, 299. 1887.

which is obtained over the entire field when parallel light is used. On turning the analyzer each ring is decomposed into four arcs, which widen when the direction is that proper to the section and contract when it is the opposite.

In sections oblique to the axis essentially as described p. 116.

In sections parallel to the optic axis essentially the hyperbolic curves of Fig. 243.

Determination of Optical Characters.

Most of the determinations are exactly as described under the preceding division. References may therefore be made as follows; *Direct determination of principal indices*, p. 103; *Extinction directions*, p. 117; *Faster and slower ray*, p. 118; *Retardation*, p. 118 *Strength of double refraction*, p. 119.

DETERMINATION OF DIRECTION OF ROTATION OF PLANE OF VIBRATION.

In a section normal to the optic axis and with crossed nicols:

(a) *By the widening of the arcs* when the analyzer is turned in the direction of rotation.

(b) *A quarter undulation mica plate* inserted diagonally above

FIG. 253.

FIG. 254.

the plate converts the rings and cross into two interwound spirals, which start near the centre and wind in the direction of rotation,* Fig. 253, Right; Fig. 254, Left. If the mica is placed below the plate the spirals are reversed.

*A plate of left and another of right-handed substance give a fourfold spiral in which the direction of winding conforms to the lower plate.

(c) *The direction in which it is necessary to turn the analyzer* to change the tint of passage (see below) into red is the direction of rotation.

DETERMINATION OF ANGLE OF ROTATION OF PLANE OF VIBRATION.

In a section normal to the optic axis and with crossed nicols:

(a) *With monochromatic light* the rotation of the analyzer necessary to produce darkness is la.

(b) *With white light and Tint of Passage or Sensitive Tint.* When the analyzer is at right angles to the brilliantly coloring yellow green, for which $\lambda = 550\,\mu\mu$ in air, the remaining colors unite to an easily recognized grayish violet, called the tint of passage or sensitive tint, because the slightest further rotation in the one direction gives red and in the other blue. Starting with the polarizer and analyzer parallel, the angle turned by the analyzer to produce the tint is la for $\lambda = 550\,\mu\mu$ (middle yellow). Fig. 255 shows the rotations produced by a quartz plate of 3.75 mm. thickness. With parallel nicols the vibration plane of yellow is then at right angles to PS, the vibration plane of the analyzer, that is yellow is extinguished and the *tint of passage* results. With crossed nicols the tint of passage results when the vibration plane for yellow is PS, that is, for a rotation of 180° or a thickness of 7.50 mm.

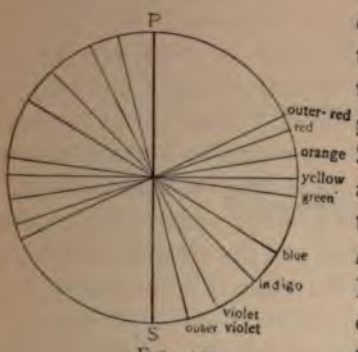


FIG. 255.

(c) *With white light and spectrum.*

The light polarized in P, Fig. 256, passes through the substance S, the collimator C, the analyzer A and through a slit and is decomposed by the prism R and viewed by the telescope T. As the analyzer is turned a dark line corresponding to the extinguished ray will move along the spectrum and the angles corresponding to the rotations (from crossed position) which bring this line into coincidence with the Frauenhöfer lines will correspond to la for the corresponding values of λ.

If a quartz plate is placed at O, the light passes through the quartz, analyzer, slit and decomposing prism and yields a spectrum

with a dark line. The analyzer is turned until the dark line coin-

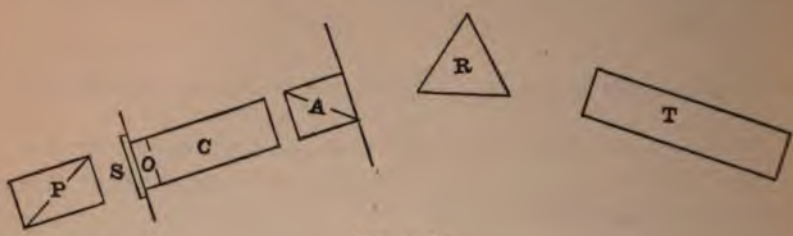


FIG. 256.

cides with a Frauenhöfer line; then the substance is introduced and the rotation necessary to bring the two lines again into coincidence is the value of la for that color.

FIG. 257.

If a biquartz,* Fig. 257, of 7.50 mm. thickness be used, with crossed nicols, or one of 3.75 mm. with parallel nicols, a black line will appear for yellow ($\lambda = 550\,\mu\mu$), which for slight rotations of the analyzer will move into the blue or the red. With the introduction of the substance at S the black line is disturbed and the rotation necessary to re-establish it is the value of la for yellow.

ABSORPTION AND PLEOCHROISM.

In uniaxial crystals with monochromatic light the two rays corresponding to any direction of transmission are in general absorbed at a different rate. The faster ray may be either more absorbed or less absorbed than the slower, the absorption of the ordinary ray being independent of the direction of transmission, while that of the extraordinary varies with the inclination to the optic axis, but is constant for the same angle and differs most from that of the ordinary for transmission normal to the optic axis.

With white light the color seen in any direction will be due to a combination of the ordinary and extraordinary rays, and this combination will give different tints in different directions.

Usually the two rays are viewed separately by means of a polarizing microscope with the analyzer out. The color varies as the

* Two semicircular basal plates of right and left quartz, of equal thickness (Biot's 3.75 mm., Soleil's 7.50 mm.) and with their twinning plane either parallel or at right angles to the vibration plane of the polarizer.

stage is turned and the maximum color differences are obtained when the extinction directions coincide with the vibration plane of the polarizer (shorter diagonal).

In sections normal to the optic axis the color is constant for all positions of the stage.

In sections parallel to the optic axis. If the extinction direction parallel to the optic axis is placed first parallel then at right angles to the short diagonal there will be seen first the extraordinary, second the ordinary.

In all other sections the extraordinary will be found to approach the constant tint of the ordinary as the sections become more nearly perpendicular to the optic axis.

The colors of ordinary and extraordinary may be contrasted side by side by means of a "dichroscope" consisting, Fig. 258, of

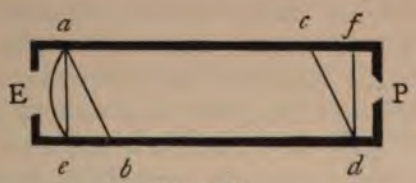


FIG. 258.

a rhomb of calcite in which *ab* and *cd* are the short diagonals of opposite faces. To these faces glass wedges *aeb*, *dfc* are cemented and the whole encased. The section is placed at *P* and the light from the substance passes through a rectangular orifice, a double image of which is seen by the eye at *E*.

When the planes of vibration of the section and the calcite coincide the two rays originated in the section will have the maximum color difference of the section.

If the plate is turned around the axis of the dichroscope each ray from the crystal is decomposed in the calcite and contributes a portion of its intensity to each of the two images and at 45° each will contribute half of its brightness to each image; that is, the the two images will then be alike in color and brightness.

Basal sections of dark tourmaline or of magnesium platinocyanide show phenomena analogous to the absorption tufts described under biaxial absorption.

CHAPTER X.

THE OPTICALLY BIAXIAL CRYSTALS.

In any orthorhombic, monoclinic or triclinic crystal the ray surface for light of any definite wave-length is symmetrical to three planes at right angles to each other which intersect in the vibration directions of the fastest and slowest rays, the third intersection being the vibration direction of a ray of some intermediate velocity.

The planes of symmetry are called *Optical principal sections* and their intersections are called *Principal vibration directions.** No diameter is an axis of isotropy, that is a direction about which the crystals are optically equivalent, but in two directions there is single refraction and these directions from analogy with the uniaxial are called the "Optic axes" *for light of that wave-length.*

As in the uniaxial crystals the relative values of the *principal indices of refraction*, that is, the indices of the rays with the principal vibration directions, change with the wave-length. But as a consequence the directions of the optic axes change also, the amount of change or so called "dispersion" varying from a few minutes to over forty degrees. That is, the optic axes in this group of crystals are not fixed in position as in the uniaxial.

With decreasing geometric symmetry not even the optical principal sections or principal vibration directions remain fixed in direction, but alter with the change of light. Upon this so-called "Dispersion of the Bisectrices" are based optical distinctions between the systems.

In *Orthorhombic Crystals* the geometric axes are always principal vibration directions and the axial planes are optical principal sections for all colors.

In Monoclinic Crystals the ortho axis *b* is always a principal vibration direction and the clinopinacoid 010 is an optical principal section for all colors, the other two principal vibration directions lie in 010, but vary in position with the wave-length.

*Often called axes of elasticity.

In Triclinic Crystals there are no principal vibration directions or principal sections which are constant for all colors.

THE OPTICAL INDICATRIX.

The inductive method by which the present conception of a biaxial ray surface has been reached may be briefly outlined as follows:

It was shown by Huyghens that the ray surface of calcite was a double surface composed of a sphere and an ellipsoid of revolution, and the ray surfaces of other tetragonal and hexagonal substances were found to be similar and symmetrical always to the crystallographic planes of symmetry. It was later shown that from an ellipsoid of revolution all the relations between direction of transmission, velocity and vibration direction could be deduced.

By analogy it was conceived that for any less symmetrical crystal some correspondingly less symmetrical figure or Optical Indicatrix existed from which these relations and the shape of the ray surface could be geometrically determined. Experiment has proved this to be true, and that for orthorhombic, monoclinic and triclinic crystals the optical indicatrix is an ellipsoid the three rectangular axes of which are in length proportionate to the so-called principal indices of refraction.

Let $\alpha < \beta < \gamma$ be these principal indices of refraction and $\mathfrak{a}$, $\mathfrak{b}$ and $\mathfrak{c}$ the *directions* of vibration of the corresponding rays, that is, of the rays with greatest, intermediate and least velocity of transmission. Upon the directions $\mathfrak{a}$, $\mathfrak{b}$, $\mathfrak{c}$ lay off $O\mathfrak{a} = \alpha$, $O\mathfrak{b} = \beta$, and $O\mathfrak{c} = \gamma$ and upon these as axes construct an ellipsoid that is the Optical Indicatrix, Fig. 259. Then, by analogy with the uniaxial, p. 102, for any diameter considered as a direction of transmission there are two points of the surface the normals from which are also normal to that diameter. *These normals are at once the directions of vibration and the reciprocals of the velocities of the rays transmitted in the direction of the diameter.*

FIG. 259.

RAY SURFACE.

The shape of the ray surface may be judged from the shape of the optical principal sections $\mathfrak{ab}$, $\mathfrak{cb}$ and $\mathfrak{ac}$.

Optical Principal Section ab. For the direction a*O*, Fig. 259, the two normals are b*O* and c*O* corresponding to velocities $\frac{I}{\beta}$ and $\frac{I}{\gamma}$; for the direction b*O* the normals are a*O* and c*O*, corresponding to velocities $\frac{I}{\alpha}$ and $\frac{I}{\gamma}$, for any other direction one normal is c*O*, the other a line in ab, found as described on p. 102 and varying in length between a*O* and b*O*, according to the direction of the ray, like the radius vectors of an ellipse. Hence for any direction of transmission one ray vibrates in some direction in ab, but is transmitted with a velocity varying like the radius vectors of an ellipse between $\frac{I}{\alpha}$ in the direction b to $\frac{I}{\gamma}$ in the direction a, Fig. 260, and corresponds to an extraordinary ray.

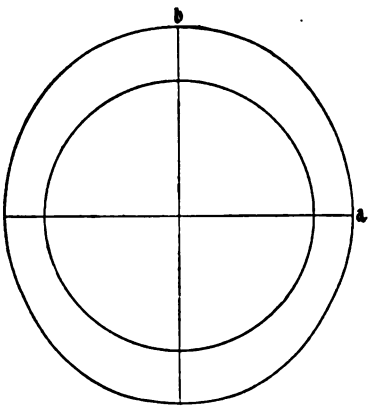


FIG. 260.

The other ray for any direction vibrates in the direction of c, normal to ab and is transmitted with constant velocity $\frac{I}{\gamma}$. It corresponds, therefore, to an ordinary ray. Hence for ab the sections of the two shells are as in Fig. 260.

Optical Principal Section, cb. In precisely the same manner it is shown that for any direction of transmission in the plane cb one ray is extraordinary in that its vibration direction is always b and its velocity varies between $\frac{I}{\gamma}$, in the direction b, to $\frac{I}{\beta}$ in the direction c; whereas the other ray is ordinary with a constant vibration direction a normal to cb and a constant transmission velocity $\frac{I}{\alpha}$. That is for cb the sections of the two shells are as in Fig. 261.

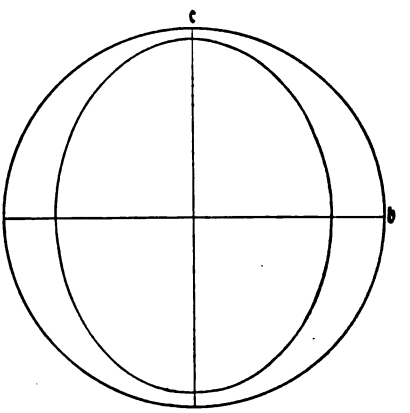


FIG. 261.

Optical Principal Section ac *or Plane of the Optic Axes.* In this case for any direction of transmission one ray is extraordinary with its vibration direction in ac and velocity varied between $\frac{1}{\gamma}$ in the direction a and $\frac{1}{\alpha}$ in the direction c, while for the ordinary ray the constant vibration direction is b and the constant velocity $\frac{1}{\beta}$. That is, in this section of the two shells, there must be four symmetrically placed points of intersection *E*, as shown in Fig. 262.

FIG. 262.

Each principal section then consists of an ellipse and a circle. The entire surface may be generated as follows.

Revolve the section ac about one of its axes a, the curves of the section touching always those in the section ab and altering according to a definite law until after 90° rotation they coincide with the curves of the section .

OPTIC AXES.*

In Section ac of the indicatrix, Fig. 259, there must be a radius vector *OL* equal β, for these vary between α and γ. Similarly *OL'* (not shown) symmetrically opposite *OL* will equal β. The sections of the indicatrix through *OL Ob* or *OL' Ob* must be circles.

In section ac of the ray surface, Fig. 262, near but not at the points *E*, common tangent planes can be drawn to each shell.

The directions normal to these circular sections in the indicatrix and to the common tangent planes in the ray surface are the so-called optic axes *AA* of Fig. 262.

A circular section of the indicatrix is the common ray front of all the diameters determined, p. 133, by the normals to the indicatrix from each point of the circle. Of these normals only one, *Ob*, is *in* the circle, hence only one diameter is normal to the circle (the optic

* The directions *EE* are variously called Ray Axes, Secondary Axes, and Biradials and are simply directions of equal ray velocity, the ray, however, corresponding to an infinite number of planes tangent at *E*, the normals to which diverge and form a cone (outer conical refraction).

axis), while the others starting from O diverge and form a cone. Furthermore since the normals are all in different directions the rays, transmitted in the directions of the diameters; have all directions of vibration. The same facts are evident from Fig. 262; the common tangent plane caps the conical depression at E. From each point of the circle of contact the lines to O represent rays with a common ray front and of which the front normal is the optic axis. That is, each optic axis is the front normal for a diverging cone of rays (inner conical refraction) which on emergence are parallel to this front normal but each of which has a different direction of vibration.

The velocity in the direction of an optic axis is evidently $\frac{1}{Ob} = \frac{1}{\beta}$.

Positive and Negative Ray Surfaces.

In the plane of the optic axes $\mathfrak{a}$ and $\mathfrak{c}$ bisect the angles between these axes. If the acute bisectrix is $\mathfrak{c}$, ($Bx_a = \mathfrak{c}$), that is, if bisects the acute angle between the optic axes the ray surface is said to be *positive*.

If the acute bisectrix is $\mathfrak{a}$, ($Bx_a = \mathfrak{a}$), that is, if $\mathfrak{a}$ is the acute bisectrix and $\mathfrak{c}$ the obtuse, the ray surface is said to be negative. This conforms strictly to the usage in the uniaxial which is a special case of biaxial with the angle between the optic axes zero, that is with one optic axis in the direction c.

Refraction in Biaxial Crystals.

In general, following the Huyghens construction, p. 86, the new ray fronts through the point E, Fig. 208, are tangent to the shells in points which do not lie in the plane of incidence, that is, both rays are bent out of the plane or are *extraordinary*. With normal incidence this is still true. Let $SURT$, Fig. 263, be the section of the indicatrix normal to the ray. The planes ROf and TOf through the axes of the ellipse and the front normal contain the two rays Oe and Ot at right angles to the surface normals RE and UN. These normals are the vibration direction of the rays and are not

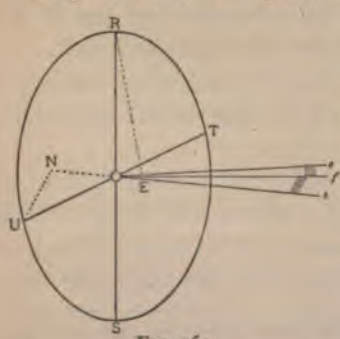


Fig. 263.

at right angles to each other, though on emergence both rays retake the direction *Of* with vibrations at right angles and a phase difference.

Because *SURT* is normal to the incident ray it is parallel to the plate, hence the extinction directions in the plate are *OR* and *OU* the projections of the vibration directions.

FIG. 264.

If the plane of incidence be a principal section then the points of tangency of the new ray fronts will be in the plane. One will for all directions show a constant refraction index and may be called the ordinary.

WITH PARALLEL MONOCHROMATIC LIGHT AND CROSSED NICOLS.

In sections normal to an optic axis the field will maintain a uniform brightness during rotation of the stage, because p. 136, in the emerging cylinder of rays developed by inner conical refraction each ray vibrates in a different plane.

In all other sections the field is dark every 90° and is illuminated for all other positions and most brilliantly in the diagonal positions. The field is also dark throughout an entire revolution for $\frac{\Delta}{\gamma} = 1, 2, 3$, etc., p. 111, and brightest for $\frac{\Delta}{\gamma} = \frac{1}{2}, \frac{3}{2}, \frac{5}{2}$, etc., in all conforming to similar uniaxial sections.

Relation between Extinction Directions and Optic Axes. As just explained, the extinction directions are the diameters *TT* and *HH*, Fig. 264, of that central section of the indicatrix which is parallel to the plate. The two circular sections must intersect any central section in *equal* diameters *tt* which from the nature of an ellipse must be symmetrical to the axis of the ellipse, that is, to the extinction directions.

If planes be passed through the plate normal and each optic axis their traces *ee* will be at 90° to those of the circular sections, hence also symmetrical to the extinction directions. *That is, the extinction directions must* bisect the angle between the traces of the two planes, each through the plate normal and an optic axis.*

WITH PARALLEL WHITE LIGHT AND CROSSED NICOLS.

In sections normal to an optic axis for light of one color there is

* Upon this principal and after a device by Prof. Groth, Böhm and Wiedemann, of Munich, construct a model for graphic determination of extinction directions.

constant illumination throughout rotation because of internal conical refraction p. 136; the optic axes for other colors are, however, more or less oblique to the section and double refraction takes place, and as the resulting vibration directions are somewhat differently oriented for each color, color tints result somewhat as in circular polarization.

In all other sections. If there is no marked dispersion of the principal vibration directions there will be approximately perfect extinction every 90° and interference colors as in the uniaxial, but with marked dispersion the vibration directions in any plate diverge for the different colors and the plate can never be perfectly dark and moreover in any position the interference colors due to phase difference are modified by color tints due to the partial or complete extinction of certain colors.

WITH CONVERGENT MONOCHROMATIC LIGHT AND CROSSED NICOLS.

In sections normal to a *or* c there will be darkness whenever the vibration planes of the emerging rays coincide with those of the nicols and whenever $\frac{\Delta}{\lambda}$ is a whole number; see p. 111. The points of emergency of the optic axis will therefore be dark, since $\frac{\Delta}{\lambda} = 0$, and the points corresponding to $\frac{\Delta}{\lambda} = 1$ will together form a ring around each axis and similarly for values of 2, 3, 4, etc., until the pair corresponding most nearly to the value $\frac{\Delta}{\lambda}$ for the centre of the field* unite at or near the centre to a cross loop or figure eight around both axes and subsequent rings form lemniscates around this as in Fig. 265. If $\frac{\Delta}{\lambda}$ for the centre is less than unity even the first ring must surround both axes, giving them a figure more

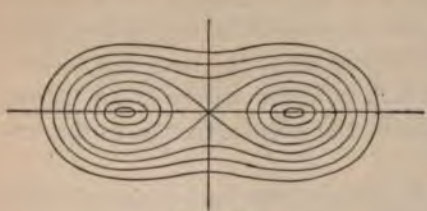


FIG. 265.

* For the rays transmitted normally at the center of the field $\frac{\Delta}{\lambda} = \frac{t}{\lambda}(n_1 - n)$. If the direction of transmission is c then $n_1 - n = \gamma - \beta$ and if direction is a then $n_1 - n = \beta - a$. In either case $\frac{\Delta}{\lambda}$ is known and determines the number of rings between the axes and the centre.

similar to that of a uniaxial crystal. As all parts of the lemniscates are independent of positions of planes of vibration, there will be no change during rotation of the stage.

When the stage is rotated so that principal sections are parallel to the vibration planes of the nicols all the rays transmitted in those sections (therefore vibrating in or at right angles to those sections) will be extinguished and there will be a sharp dark band joining the poles and another somewhat thicker lighter band at right angles to the first and midway between the poles. These are often called brushes. For any other position of the stage the vibrations of some other rays are parallel to those of the nicols, and these are always so distributed that the resultant dark spots form the branches of an hyperbola through each of the axes. At the diagonal position the effect is as in Fig. 266.

FIG. 266.

As the stage is rotated the straight bars appear to dissolve into the hyperbola, the branches of which appear to rotate in the opposite direction to the stage, the convex side always toward the other axis. If $\frac{\Delta}{\lambda}$ is nowhere equal to 1 there will be no dark rings, but the hyperbola will appear as before. Because the two principal sections are planes of symmetry the curves and brushes are at all times symmetrical to the traces of the sections.

In sections normal to b the dark curves due to $\frac{\Delta}{\lambda} = 1, 2, 3,$ etc., are hyperbolæ similar to those shown, Fig. 243. No brushes are visible.

In sections normal to an optic axis there will be the same formation of dark rays corresponding to whole numbers for $\frac{\Delta}{\lambda}$, the curves, however, will be more nearly circles, Fig. 267. There will be a straight black bar bisecting the curves whenever the trace of the plane of the optic axes coincides with the vibration direction of either nicol, and for all other angles of rotation there will be

one arm of an hyperbola through the axis, Fig. 268, the convex side towards the other axis. This arm will rotate in the opposite direction to the stage.

FIG 267.

FIG. 268.

WITH CONVERGENT WHITE LIGHT AND CROSSED NICOLS.

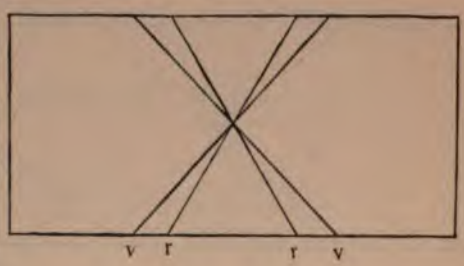
In sections normal to the acute bisectrix Because of the dispersion of the optic axes described, p. 130, the centres of the ring systems are not the same for the different colors. If there is also a dispersion of the bisectrices the section can only be exactly normal to the bisectrix for one color. These facts, added to the general one that for different colors the greater the wave-length the greater the distance apart of the rings, render it possible in white light to determine the crystalline system by the distribution of the colors, thus obtaining Divisions V, VI and VII.

V. ORTHORHOMBIC CRYSTALS.

The color distribution must be symmetrical to the line joining the optic axes and to the line through the centre at right angles thereto, because these are the traces of the principal sections common to the ray surfaces of all the colors. Assuming that the same principal section is the axial plane for all colors there may be two cases: 1°. The axes for the longer wave-lengths may be most dispersed, that is, the angle between the optic axes for red greater than that for violet, or $\rho > \upsilon$. 2°. The axes for the shorter wave-lengths may be most dispersed, that is, the angle between the optic axes for red less than that for violet, or $\rho < \upsilon$.

Figure 269 illustrates the second class. In the interference figure for the diagonal position the hyperbolae will pass through the axial points for each color and in each that color will be extinguished, giving, if the dispersion is great, a series of colored bands ranging from red (white minus violet and part of blue)

through the violet axes; to blue or violet (white minus red and part of yellow) through the red axes. Generally the intermediate colors overlap, producing a dark band, except on the outer fringes. That is, the colors* fringing the hyperbola are in inverse position to the axial points.



Similarly, if the full line circles represent the first extinction ring for violet and the dotted those for red, then as we pass from the centre red is first extinguished and the violet tint shown.

For $\rho < \upsilon$ the red is further from the centre than the blue. For $\rho > \upsilon$ the red is nearer the centre than the blue. Or in general, the color with the larger angle is nearer the centre of the field.

If the axial plane is not the same for all colors, that is, if b for certain colors becomes a or c for others, then the interference figures will be turned 90° with respect to each other and, if superposed in white light, will give a complex figure as in brookite.

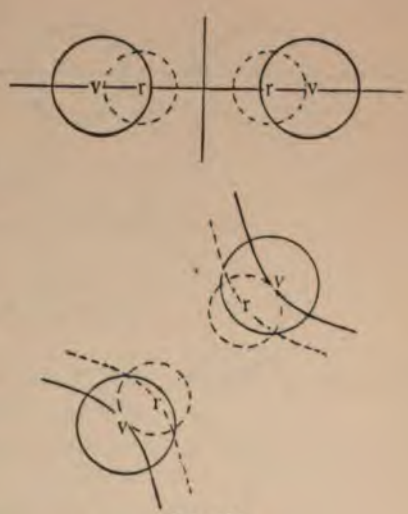


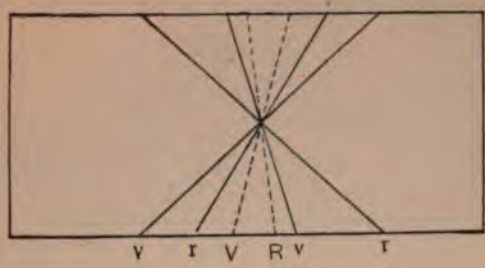
FIG. 269.

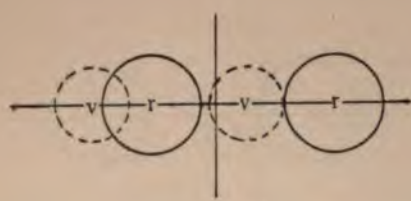
VI. MONOCLINIC CRYSTALS.

There must also be considered the dispersion of the bisectrices. Three general cases have been distinguished, in all of which 010 is a principal section and the ortho axis *b* a principal vibration direction.

* The dotted curves should be colored blue; the full line curves red, in Figs. 269–272.

(a) *Inclined Dispersion.* The optic axes for all the colors lie in the plane 010 and the ortho axis b is $\mathfrak{b}$, the intermediate principal vibration direction, but $\mathfrak{a}$ and $\mathfrak{c}$ vary in position in the plane 010. Evidently the color distribution will be symmetrical to 010.





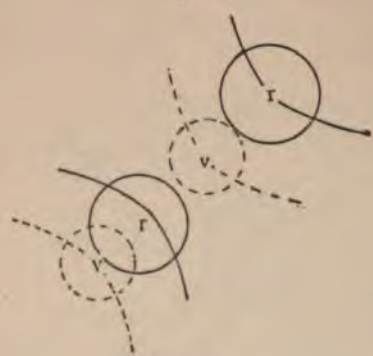


FIG. 270.

Let Fig. 270 represent the plane 010 in a section cut normal to the acute bisectrix for yellow light. The acute bisectrices for red and violet will be oblique to this section as represented. Assume $\rho > \alpha$ and construct the interference hyperbolæ and first ring for the diagonal positions. Four things are at once noticed:

1°. The color-distribution and shape are symmetrical to the trace of 010 and to this line* only.

2°. The combination color rings around the axes due to overlapping of rings for the different colors are very different, the one relatively small, circular and intense, the other larger, oval and relatively dull. The sequence of colors may be the same or reversed.

3°. The smaller ring is further from the centre of the field.

4°. One hyperbola band is much broader than the other.

(b) *Horizontal Dispersion.* If the ortho axis b is the acute bisectrix (either $\mathfrak{a}$ or $\mathfrak{c}$ as the crystal is + or −) then the planes of the optic axes must for all colors pass through it and be normal to the plane 010. In a section cut normal to the acute bisectrix for yellow light let V and R, Fig. 271, represent the axial planes for violet

* Line connecting the axial points.

and red, *rr* the optic axes for red, *vv* those for violet. The plane 010 (not shown) is normal to all the axial planes.

In the interference figure it is evident that the color-distribution and shape of the lemniscates are symmetrical to the trace* of 010 but not to any other line. Hence, for normal position the hori-

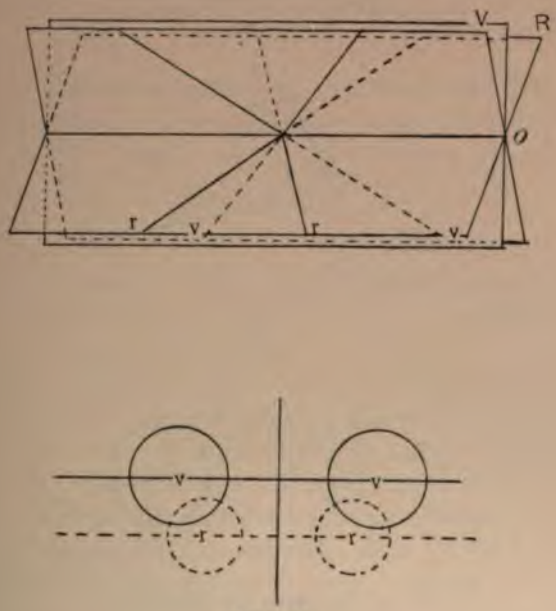


FIG. 271.

zontal dark bar will be centrally black from overlapping colors, but on the edges will be red and violet or blue.

(c) *Crossed Dispersion.* If the ortho axis *b* is the acute bisectrix (either c or a as the crystal is positive or negative) then will the section normal to the acute bisectrix for one color be normal for all and will itself be parallel to 010. The axial planes for all colors will pass through *b*, and as *b* corresponds to a or c it is an axis of binary symmetry, hence the color-distribution and shape of the lemiscates must be such that any straight line through the centre of the field passes in each direction through similar points. That is, diagonally opposite parts of the field are similar. In Fig. 272 *V* and *R* are the axial planes for violet and red.

It will be noticed that the chief reliance throughout is upon the

*Normal at centre to line joining the axes.

colors that fringe the horizontal brush or the hyperbola, and that this is in every case the other end of the spectrum to the color of the light producing it.

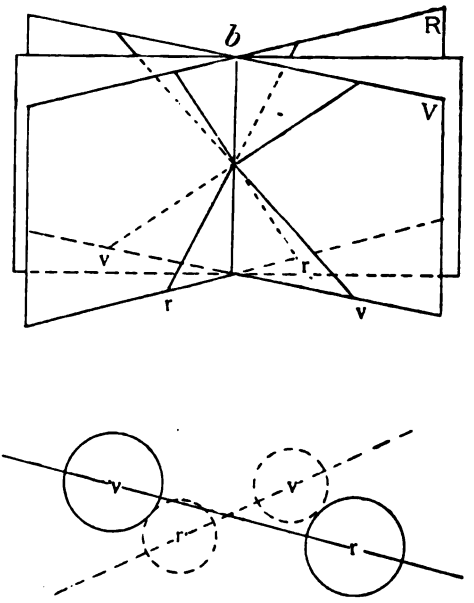


FIG. 272.

VII. TRICLINIC CRYSTALS.

With triclinic crystals there is no symmetry at all in the arrangement of the colors in the interference figure from white light. Two or more varieties of dispersion may occur at once.

Plates normal to the obtuse bisectrix show similar phenomena, though frequently it is impossible to obtain the interference figures within the field. The color-distribution is naturally changed as the dispersion of axes is in the opposite direction.

CHAPTER XI.

DETERMINATION OF THE OPTICAL CHARACTERS OF BIAXIAL CRYSTALS.

Some of the determinations are exactly as described under the uniaxial. References may therefore be made of follows: *Faster and slower ray*, p. 146; *Retardation*, p. 146; *Strength of double refraction*, p. 147; *Thickness of section*, p. 147.

Determination of Planes of Vibration or "Extinction."

In biaxial crystals complete extinction in general is only obtained with monochromatic light, p. 138. The methods of determination are as described, p. 137.

In sections normal to one of a, b, or c the extinction directions are the other two principal vibration directions. In sections parallel to a, b, or c one extinction direction must be a principal vibration direction. In sections normal to an optic axis no extinction directions are obtained. The general relation between extinction directions and optic axes is described, p. 137. This test is often sufficient to determine the crystalline system, especially when an entire crystal can be examined and the extinction directions determined in all faces and zones.

In orthorhombic crystals extinction will always be *parallel or symmetrical* to crystallographic edges, cleavage cracks, etc. In pinacoidal faces the vibration direction will be parallel to the crystal axes.

In monoclinic crystals there will be parallel or symmetrical extinction in the zone [001 100] and in this zone the vibration directions in the base and orthopinacoid will be parallel to the axes. In all other zones the extinction will be *oblique*.

In triclinic crystals all extinctions will be oblique.

The Orientation and Distinction of the Principal Vibration Directions.

If the positions of the optic axes are known then all three principal vibration directions a, b and c are also known, for a and c are

the bisectrices of the angles between the optic axes and $\mathfrak{b}$ is normal to their plane. The indices α, β and γ may then be determined, as described later, and the indicatrix and ray surface therewith constructed.

In orthorhombic crystals the pinacoids are normal to $\mathfrak{a}$, $\mathfrak{b}$ and $\mathfrak{c}$. That normal to $\mathfrak{b}$ will show with convergent monochromatic light hyperbolæ* similar to Fig. 243 and those normal to $\mathfrak{a}$ and $\mathfrak{c}$ will show lemniscates, Fig. 270.

In monoclinic crystals the clinopinacoid is necessarily either normal to $\mathfrak{b}$ and shows the hyperbolæ, Fig. 243, with convergent monochromatic light, or is normal to $\mathfrak{a}$ or $\mathfrak{c}$ and shows lemniscates. The extinction directions therein are the other two principal vibration directions, and the faster and slower ray may be distinguished as described, p. 146. Plates may be ground normal to these directions.

In triclinic crystals by trial a face is found which with convergent monochromatic light shows, even eccentric, the hyperbolæ or lemniscates or axial image; from this the direction is judged in which a new face may be ground exactly† normal to a principal vibration direction. The extinction directions in the new face will be the other principal vibration directions and the faster and slower rays corresponding may be distinguished as on p. 146. Sections may be ground normal to them. If there is dispersion of bisectrices accurate work will require special sections for each color, if the dispersion is not large, sections for a middle color, usually soda yellow, are used.

Direct Measurement of the Principal Indices of Refraction.

(*a*) Prisms with a refracting edge B, Fig. 212, parallel to a principal vibration direction will give for minimum deviation, p. 88, a direction of transmission RS so lying in an optical principal section that the ray vibrating parallel‡ to the edge B will yield a principal index. In general the other obtainable index is not a principal index.

By Fig. 259 it is seen that if B is parallel to $\mathfrak{c}$ the transmission

* By means of one of the attachments to the polarizing microscope described later, sections approximately normal to $\mathfrak{a}$, $\mathfrak{b}$ or $\mathfrak{c}$ may be tipped so that the lemniscates or hyperbolæ are obtained.

†It is important to leave faces of known forms so that later the ground faces may be goniometrically oriented.

‡The ray which penetrates a nicol when the short diagonal is parallel to B.

is in $\mathfrak{ab}$ and the ray vibrating parallel to B yields γ. Similarly if B is parallel to $\mathfrak{b}$, β results; and if B is parallel to $\mathfrak{a}$, α results.

If the bisector BD, Fig. 212, is also the trace of an optical principal section then the direction of transmission RS for minimum deviation is also a principal vibration direction and both rays yield principal indices.

(*b*) Prisms in which BD, Fig. 212, the bisector of the refracting angle B is a principal vibration direction also give for minimum deviation a direction of transmission RS lying in an optical principal section. Therefore the index of the ray vibrating parallel to BD, that is at right angles to the edge B, is a principal index.

By Fig. 259 it is seen that if BD is parallel to $\mathfrak{c}$ the transmission RS being normal to BD lies in the plane normal to $\mathfrak{c}$, that is in the optical principal section $\mathfrak{ba}$ and the ray vibrating parallel to BD or at right angles to B will yield γ. Similarly if BD is parallel to $\mathfrak{b}$ the ray vibrating at right angles to B will yield β and if BD is parallel to $\mathfrak{a}$ the ray vibrating at right angles to B will yield α.

If $\mathfrak{a}$, $\mathfrak{b}$, or $\mathfrak{c}$, bisect the exterior angle B, Fig. 212, then is the bisector BD normal to this principal vibration direction and RS parallel thereto, that is for minimum deviation the direction of transmission is a principal vibration direction and both rays yield principal indices. The formulæ under (*b*), p. 103, hold for (*a*) and (*b*).

(*c*) If one face AB, Fig. 228, of a prism is an optical principal section and the refracting edge A is a principal vibration direction, rays normally incident at AB will be transmitted in the direction RS parallel to another principal vibration direction and each ray will yield a principal index. If AB is $\mathfrak{ab}$ and A is either $\mathfrak{a}$ or $\mathfrak{b}$, Fig. 259, then RS is $\mathfrak{c}$ and α and β result. The formulæ under (*c*), p. 103, hold good.

(*d*) In any crystal face or section parallel to a principal vibration direction, by the methods and formulæ of pp. 90–95, if this direction is made the direction of transmission the two limit lines, as in p. 104, correspond to principal indices; for instance if the transmission direction is $\mathfrak{a}$ the two limit-lines correspond to β and γ. If the plate is turned 90° in its own plane the direction of transmission will be normal to a principal vibration direction (to $\mathfrak{a}$ in case given) and the limit-line of the ray the vibration direction of which is at right angles to this direction of transmission will correspond to the third index.

If the plate is parallel to a principal section* then either principal vibration direction used as a direction of transmission will give two of the principal indices.

DETERMINATION OF THE ANGLE BETWEEN THE OPTIC AXES.

This is usually accomplished in a plane parallel plate cut normal to the acute bisectrix. In orthorhombic crystals the same plate will be normal for all colors, but in the other systems this is not so, but if the plate be cut normal for a middle color, say yellow, the results for all colors will be approximately† accurate. The apparent angle,‡ denoted by $2E$, is larger than the true angle, denoted by $2V$, and, since light transmitted in the direction of the optic axis has the middle index β, p. 136, then

$$\beta = \frac{\sin E}{\sin V} \text{ or } \sin V = \frac{\sin E}{\beta}.$$

If $2E$ exceed 180° the plate is usually surrounded by oil or some other dense medium in a vessel with plane parallel sides. Denoting the angle then obtained by $2H$ and the index of the surrounding medium by n

$$\sin V = \frac{n \sin H}{\beta}.$$

By measuring a second plate normal to the obtuse bisectrix there results

$$\sin V' = \frac{n \sin E'}{\beta}$$

but as $2V + 2V' = 180°$ then $V + V' = 90°$ and $\sin V' = \cos V$ whence

$$\frac{\sin V}{\cos V} = \frac{n \sin E\beta}{n \sin E'\beta} \quad \text{or} \quad \tan V = \frac{\sin E}{\sin E'}$$

*If the plate be parallel to the optic axes the two boundaries approach as the plate is turned and meet at the optic axis. In fact the shape of both sections of the Fresnel's surface may be plotted and even the optic axial angle measured. Liebisch, *Grundriss der Phys. Kryst.*, 373.

† The rays can only emerge at the true angle when the optic axes are normal to the surface, as in a sphere or in a cylinder the axis of which is perpendicular to the axial plane; or when the outer medium has the same index of refraction as the ray transmitted in the direction of the optic axis. The former is rarely practicable, the latter will be described later.

‡ Plates normal to obtuse bisectrix yield $V' = 90° - V$.

This measurement involves actual rotation of the plate about b as an axis between the lenses of a polariscope, the plane of rotation being at 45° to the vibration planes of the crossed nicols and the arms of the hyperbola being brought in succession into tangency with one cross hair. The line joining the optic axes in the images must coincide with the other cross hair.

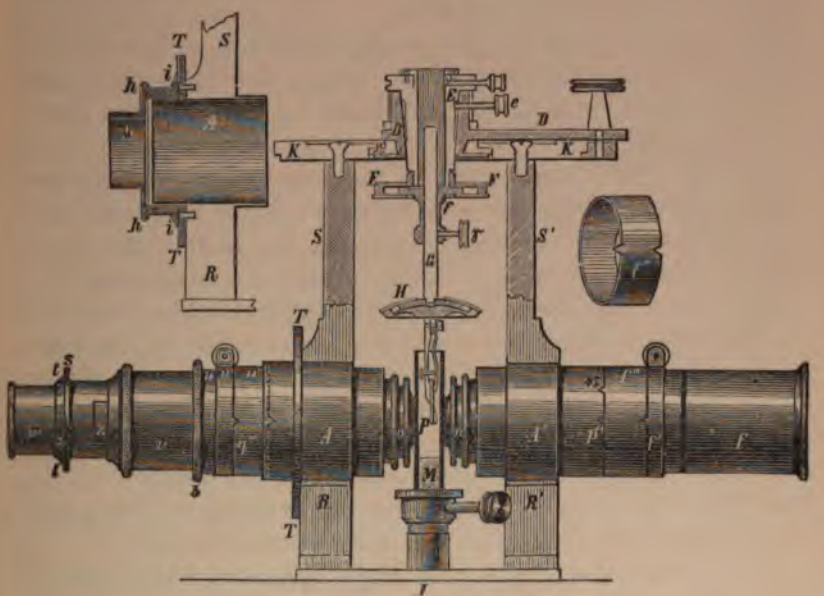


FIG. 273.

The Groth Universal Apparatus. The optical portion of the polariscope shown, Fig. 235, is placed in horizontal position in another stand, Fig. 273. The nicols are crossed at 45° to the horizon and just enough space left between the objectives to permit free rotation of the crystal around a vertical axis. Above is a horizontal graduated circle *K* with concentric central rings *E* and *F* which in turn support the vertical crystal carrier. The latter has motions of adjustment for the crystal plate which in the latest type of instrument are exactly those of the Fuess Goniometer, p. 67, but more frequently are simple sliding motions of a plate at *f* and spherical segment ("mushroom joint") at *H*. In this simpler type the mineral is centered by drawing the front tube back until the crystal is in focus, then centering as in the goniometer, then

the images are centered by the mushroom joint until both stay on a horizontal line.

The closer the lenses the larger the image: that is, a small crystal should be used, and as the best results are obtained with rather narrow interference rings the crystals should be of such a thickness (selected or obtained by grinding) that the lemniscates are distinct. In measuring in a denser medium* the glass vessel *M* is used and the lenses touch its walls.

It is usually a sufficiently accurate selection of a plane normal to a bisectrix if the center of the lemniscate coincide with the center of the field. Groth gives† methods for more accurate determination of the plane.

The No. 2 Fuess Goniometer, Fig. 193, may be used for the measurement of axial angles by substituting nicols for the signal tube in the collimator and for the eye piece in one of the telescopes; both nicols are adjustable. A tube containing a condensing lens is fitted over the objective end of the collimator like a cap and the telescope is converted into a weak microscope by the extra lens used to bring a crystal into focus. The crystal section is carefully mounted as previously described. The telescope is set opposite the collimator, the nicols being crossed 45° to the plane of rotation of the goniometer and rotation of the carefully centered crystals brings the two branches of the hyperbola successively into contact with the vertical cross hair.

By a combination of the axial angle apparatus with a spectroscope‡ the angle for all portions of spectra can be determined.

A Polarizing Microscope may be used, the interference figure being seen:

(*a*) By placing an extra lens§ (Klein's lens) above the eye piece, thus making visible a little image which forms there, or

(*b*) By placing below the eye piece but above the focal plane of the objective a weak magnifying lens (Bertrand lens).

Two methods of measuring are possible:

(*a*) *Measurement of Distances Apart of Emerging Axes.* The nicols are set at the diagonal positions and the distance *d* from the

*May use olive oil, α Brom-Napthalin, Methylene Iodide, etc., see p. 95.

†*Physikalische Kryst.*, 714.

‡A. E. Tutton, *Philos. Trans. R. Soc. London*, 1894, 185, 913. E. A. Wülfing, *Min. Mitth.*, 1896, 15, 49.

§Tschermak's *Min. Mittheil.*, XIV., 375.

centre to either hyperbola, or the average of the two distances, is read by either a glass- or screw-micrometer eye piece. Then $\sin E = \frac{d}{C}$, or in liquid, $\sin H = \frac{d}{NC}$, or $\sin V = \frac{d}{\beta C}$, in which C is a constant for the same system of lenses and is determined once for all with a crystal of known axial angle.

If the section is oblique to the bisectrix it may be tipped so that the axis of the microscope coincides with the direction of a bisectrix, and in this way a well defined interference figure is frequently obtained. A simple instrument for this purpose* consists of a clip supported 15 mm. above the stage by a ball and socket joint giving free motion in all directions, through an angle of about 45°. Delicacy of manipulation is secured by a long key fitting into the clip arm. A special lens combination is needed in order to bring the condenser nearer the section.

(*b*) *Measurement by Rotation of the Section as in the Universal Apparatus.* This involves some form of rotation apparatus attached to the stage of the microscope, by which the angle of rotation can be read. Two forms of one of the simpler devices are shown in Fig. 274. The crystal is fastened by wax to the end of a glass axle the rotation of which is measured on a graduated vertical circle. The crystal projects into a little glass trough filled with refracting liquid and must, by trial, be adjusted so that the plane of the optic axes is vertical. The measured angle is then 2 H. A similar apparatus is described by Bertrand.†

If the rotation necessary to bring the hyperbolæ into tangency with the cross hair cannot be secured on account of the closeness of the objective to the section; one hyperbola may be brought just into the field, then the other hyperbola to the corresponding point on the opposite side of the field. The angle of rotation necessary to install the second point, plus the angle equivalent to 2 d, the distance apart of the two points, will be 2 H.

A much more elaborate device is needed to both orient the axial plane and measure the axial angle. Perhaps the best for the examination of small complete crystals is the so-called Universal Rotation Apparatus‡ of Professor Klein, Fig. 275, which is espe-

*Described by T. A. Jaggar, Jr., *Amer. Jour. Sci.*, III., 129, 1897, and made by Bausch & Lomb Optical Co., Rochester, N. Y.

†*Bull. Soc. Min. de France*, III., 96, 1890.

‡C. Klein, *Sitzungsber. der. Akad. Wissenschaften. Berlin*, 1895, p. 91. "Der Universaldrehapparat," etc.

cially designed as an attachment to a No. VI. Fuess Microscope, p. 108. The microscope is tipped with its axis in horizontal position and the plate G of the rotation apparatus fastened to the microscope stage by strong clips so that the axle P is vertical. The crystal is attached at F by wax and centered by the cross hairs of the microscope and the rectangular and rotary movements of the microscope stage. Thereafter it may, while still in the line of sight, be rotated measured amounts in three planes at right angles to each other by the vertical arcs L and L_1, and the horizontal circle K. A small cubical vessel with two opposite walls of glass is filled with a strongly refracting liquid and raised into position between the converger and objective, and so that the crystal is immersed. If by previous examinations p. 145 or from the geometric properties any optical principal section is known, this will be placed parallel to one of the planes of rotation. If by manipulating the arcs L and L_1 with the nicols horizontal and vertical, a position of the crystal is found, such that the field remains dark throughout a complete rotation by K; a principal section is horizontal, for in no other sections would the vibrations of the refracted rays remain horizontal and vertical, p. 137. If this principal section is the plane of the optic axes, four times in the course of the rotation there will be a brightening of the field due to the emergence of an optic axis (internal conical refraction), p. 135. If this does not occur the crystal is remounted with this principal section vertical and parallel to one of arcs L, L_1. Then, if the arc is tipped and trial is made by K, another principal section must be found and

FIG. 274.

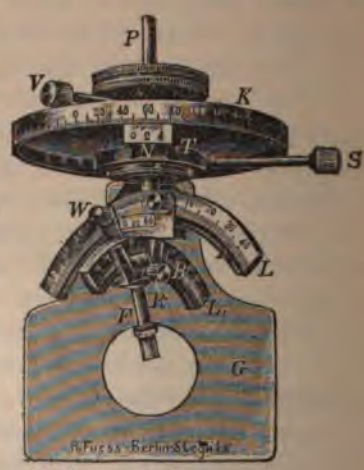


FIG. 275.

90° away the third, and one of these must be the axial plane.

Having found the plane of the optic axes the approximate positions of the axes are noted, the converger is then introduced, the nicols are set in the diagonal positions, the Bertrand lens inserted or the Klein ocular, the branches of the hyperbola are brought into tangency and the four positions of the emerging axes are read.

A special lens system is needed with this apparatus to counteract the unusual distance between crystal and objective.

Measurement of the True Axial Angle.

If the refracting liquid used in any of the preceding methods has an index of refraction equal β then sin $H =$ sin V. To practically obtain this, a liquid with n a little above β is used and the diluent added drop by drop until the boundaries of the crystals fade. Then for the accurate determination some coarse powder of the substance is placed on an object glass, a drop of the liquid added, a cover glass pressed on and the extinction directions determined in a selected grain. With the condenser lowered and the analyzer out, the two directions of extinction are successively made to coincide with the vibration direction of the polarizer, and the microscope is focussed sharply on the dark boundary between the liquid and the grain. The objective is then raised and the dark boundary line appears to move towards the substance with the higher index of refraction. If the liquid is of the mean refractive index of the crystal the border will for one position move towards the liquid and for the other towards the grain.

A final proof of the correctness of the preparation is that if $n = \beta$ $2H_a + 2H_0 = 180°$, but if on measurement $2H_a + 2H_0 < 180°$, then $\frac{n}{\beta} < 1$ and too much diluent has been added or if $2H_a + 2H_0 < 180°$ then $\frac{n}{\beta} > 1$ and more diluent should be added.

Calculation of Axial Angle from Indices.

This angle may be calculated from the principal indices of refraction by the equation

$$\tan V = \sqrt{\frac{\frac{1}{\alpha^2} + \frac{1}{\beta^2}}{\frac{1}{\beta^2} + \frac{1}{\gamma^2}}}$$

Determination of Character of Ray Surface.

This requires in general a section normal to a or c that is showing the lemniscates, Fig. 266, with convergent monochromatic light or an oblique section tipped to show these lemniscates.

The difference between the lemniscates formed normal to the acute bisectrix and the obtuse bisectrix cannot always be judged and may require measurement of the axial angle.

If the section is normal to the acute bisectrix then the latter may be proved to be c or a, that is, the Ray Surface proved to be positive or negative as follows:

(*a*) The line joining the optic axes is placed in diagonal position between crossed nicols with parallel white light the insertion of a test plate will determine as described, p. 118, whether this direction is the vibration direction a of the faster ray or c of the slower ray (the other extinction direction being b), that is, whether this section is normal to c or a and p. 136, $Bx_a = c, +$; $Bx_a = a, -$

(*b*) With convergent light and crossed nicols the line joining the axial points is placed in diagonal position and the quartz wedge, p. 118, gradually inserted with the direction c parallel to this line. If the crystal is positive the rings around each axis will expand, moving toward the centre and merging in one curve, and will contract and increase in number, if the crystal be negative, the change increasing with the thickness of wedge interposed. If the direction c of wedge and the line joining the axial points are crossed the effects are reversed;* that is expansion of the rings when the length of the wedge is parallel to the line joining the axes proves positive character and expansion when these directions are crossed proves negative character.

(*c*) If with the line joining the axial points in *normal* position the mica plate is inserted in the diagonal position the black cross is destroyed and the rings broken and new flecks developed in the same quadrants as in uniaxial, but the positions of the flecks do not so satisfactorily suggest the signs + and —

If the section is normal to the obtuse bisectrix all the results are reversed.

Crystals in Thin Rock Sections.

The proper orientation of a haphazard section may be obtained with the Jaggar apparatus, but for precise measurement a more

* Hence it is always practical to develop the lemniscate, even when only the hyperbolæ are visible, by inserting the wedge so as to produce contraction.

elaborate apparatus is essential. The different types of universal stage of von Federow* require the sections to be mounted upon a special round glass with high index of refraction, while the apparatus of Klein,† Fig. 276, permits the use of ordinary square or rectangular object glass, the cover glass, however, being removed. The metal plate G is clamped to the microscope stage. The metal vessel B, which is somewhat more than a half sphere, is filled with some liquid of high refractive index, usually glyerine $n = 1.46$.

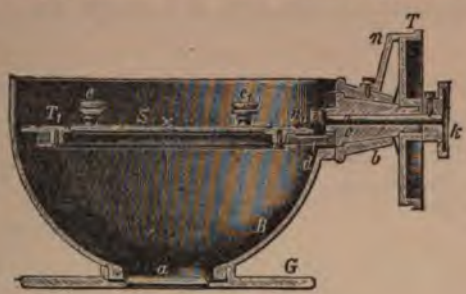


FIG. 276.

The light enters through the glass plate a. The section is supported in the liquid by the glass plate S in the rim T_1 and receives motions of rotation around the horizontal axis c by the graduated circle T, and in its own plane by the knob k, which turns the fine-toothed wheel z in contact with teeth in the rim T_1.

ABSORPTION AND PLEOCHROISM.

The vibration directions of the rays of monochromatic light which undergo greatest and least absorption and a direction at right angles to these constitute the so-called absorption axes. Their lengths are quite independent of the lengths of the axes of the optical indicatrix, but their directions are frequently though not necessarily the same. Upon these axes can be constructed an absorption ellipsoid from which the absorption in all directions can be deduced.

In orthorhombic crystals the directions of the absorption axes coincide *for all colors* with the directions of crystal axes and the axes of the optical indicatrix.

In monoclinic crystals one absorption axis is parallel to the crystal axis b, therefore to one of the axes of the optical indicatrix; the

* *Zeit. f. Kryst.* v. 25, p. 351.

† *Sitzungsber. Berlin Akad.*, 1895.

other axes lie in the plane 010, but for any color may or may not coincide with the axes of the optical indicatrix for that color.

In triclinic crystals no coincidence is essential.

Although subject to many exceptions, the law of Babinet is generally correct that "the slower transmitted ray is the more absorbed."

The relative absorption of the two rays for any direction of transmission can be obtained by use of the spectrophotometer of P. Glans, which consists, Fig. 277, of a collimator *C*, telescope *H*

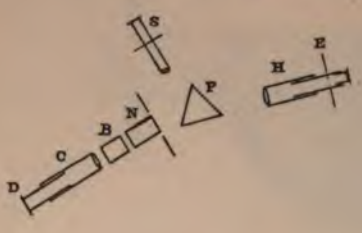


FIG. 277.

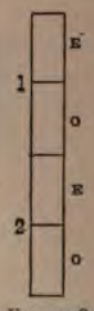


FIG. 278.

and prism *P*. The vertical slit *D* in the collimator is divided into halves by a cross piece of metal, and the entering light passes through a so-called Rochon's quartz prism *B*, which gives a double image, Fig. 278, of each half of the slit, the height of the slit being so chosen that the extraordinary image of one and the ordinary of the other are in contact in the centre of the field and the other two are shut out in the telescope. If now the crystal plate is placed in front of the slit with its extinction directions parallel and at right angles to the slit the two rays developed will pass through the prism *P* and their spectra be seen in the telescope one above the other, and by means of the horizontally movable eye plate *E* any portion may be viewed alone and compared.

By introducing a nicol *N* arranged with a graduated circle, so that when at zero its plane of vibration coincides the principal section of the Rochon prism, either ray may be shut out at will by turning the nicol to 0° or 90° and the spectrum of the transmitted ray compared with a spectrum scale sent through *S* and reflected into the telescope.

A quantitative comparison can be made as follows for any portion of the spectrum. The crystal plate is removed and the angle of

rotation α of the nicol determined at which the two spectra of the flame are equally bright; this will be somewhere near 45° but not exactly. The plate is then replaced and the nicol again turned through an angle β to secure equal intensity for the desired portion of the spectrum. The relative intensities are:

$$\frac{I_o}{I_u} = \tan^2 \alpha \cot^2 \beta.*$$

If only one of the halves of the slit is covered the intensity may be compared with that of the incident light.

Transmission along one of the principal vibration directions a, a or c will yield the intensities for vibrations parallel to the other two. As pointed out these are directions of absorption axes in the orthorhombic crystals but not necessarily in the monoclinic or triclinic crystals.

Generally speaking, the records made are simply comparative, for instance: Absorption strong $a > b > c$, or, Absorption feeble parallel a. More commonly the record is as to pleochroism or effect with white light, the color differences for vibrations parallel b, b and c for some middle color being recorded, or, the colors for directions referred to some definite crystal face.†

The pleochroic images are viewed either with a microscope or dichroscope as described, p. 131, or the apparatus of Klein, Fig. 275, may be conveniently used, after the measurement of the angle between the optic axes, by adjusting the crystal with the plane of the optic axes horizontal. If the plane of vibration of the polarizer is vertical and the analyzer is out, the color obtained throughout a rotation by K corresponds to b. If the vibration plane of the the polarizer is horizontal, transmission parallel to the bisector of axial angle yields the color for one of a or c and by a rotation of 90° with K the color of the other results.

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* Liebisch *Grundriss der Phys. Kryst.*, p. 312.

† For example: Andalusite. Pleochroism a blood red to rose red, b = c olive green. *Dana's System*, p. 496.

Titanite. a = yellowish red, b = greenish red, c = pale yellow. Ripidolite. Yellow to reddish for vibrations normal to c01, green parallel 001. *Rosenbusch Lüüfstabellen*, III e.

Axinite. Pleochroism strong, normal to r pale olive-green, giving with dichroscope olive-green and violet blue, parallel r and normal to edge r/M cinnamon brown, giving cinnamon brown and violet blue. *Dana's System*, p. 528.

ABSORPTION TUFTS.

Plates of crystals with strong absorption, especially if cut normal to an optic axis, often show in convergent non-polarized light two dark tufts symmetrical to the plane of the optic axes, but separated by a bright space. If white light is used the pencils are of different color to the rest of the field.

In Fig. 279, A is the optic axes to which the plate is normal; A' is the other optic axis. Oblique rays emerging at any point m on the line AA' have been transmitted in the plane of the optic axes and, therefore, one of the rays vibrates in the plane of the optic axis, the other parallel to the principal vibration direction b or AN. Oblique rays from any point m' on the line AN normal to AA' will vibrate (see p. 137) parallel and at right angles to the bisector of the angle $Am'A'$. As explained by Mallard† the intensity of the pencil of rays emerging at m' is necessarily less than that of the pencil of rays emerging at m, the difference increasing the nearer the points are to each other, that is, normal to AA' is a dark tuft growing lighter on each side of A. Similar effects are obtained for points not on AN, the total result being two tufts or pencils of hyperbolic shape tangent to AA'.

FIG. 279.

When white light is used if the optic axes are dispersed, the tufts are not exactly superposed and the borders show color tints.

The test may be made by holding the crystal section close to the eye and looking at the bright sky or by removing the polarizers from a converging polariscope.

If the plates are examined in convergent polarized light, the analyzer being removed, two types may be distinguished as follows:

TYPE I. The dark tufts will appear normal to the plane of the optic axis and in a clear field, when the plane of the optic axis is normal to the vibration plane of the polarizer. Example: *Andalusite of Brazil.* Plate cut normal to acute bisectrix and placed on a thin plate of azurite shows black tufts on a blue ground.

* W. Voigt zur Theorie der Absorption des Lichtes in Krystallen *Wiedemann's Ann.*, 1885, XXIII., 577.

† *Traité de Cristallographie*, II., 361.

TYPE II. The same phenomenon results when these planes are parallel. Example: *Epidote of Knappenwand Tyrol.* Cleavage plate parallel 001 gives tufts symmetrical to 010, with white light the borders towards the acute bisectrix are dark green and the opposite are dark red.

In both types, with a rotation of the plate through 90° from these positions, tufts also appear parallel to the plane of the optic axes.

METALLIC REFLECTIONS OR METALLIC LUSTRE.

The metals do not obey the usual laws of reflection; incident plane polarized light is reflected as such, only when the plane of vibration of the incident light is parallel or at right angles to the plane of reflection; and in both of these cases there is developed a change of phase, which is, however, greater in the latter case than in the former. For all other azimuths of vibration the vibration of the reflected light is the result of two unequal* components at right angles and is, therefore, elliptically polarized.

The intensity† of the reflected light differs also from that obtained with transparent substances. For normal incidence it is not zero, but considerable, decreasing as the angle of incidence increases up to a value of 50° to 60°, thereafter again increasing until the angle of grazing incidence is reached.

For a given incidence certain colors are reflected by metals more strongly than others, whereas, transparent substances, even if colored, reflect white light. The combined effect of this intense and selective reflection is called *Metallic Lustre.*

SURFACE COLORS.

Between metallic and transparent substances are certain which are transparent for light of certain wave-lengths, but stop others so completely that even with thin sections they are represented in

*Varying from 0 for normal incidence to $\lambda/2$ for grazing incidence and for a particular angle, which is experimentally found to be the angle of maximum polarization of common light, passing through the value $\lambda/4$.

†Cauchy, Voigt and Drude have studied the phenomena and calculated the indices of refraction and absorption for a number of metallic substances for different vibration directions upon fresh cleavage surfaces. For gold and silver a higher velocity of light was found than in air, but very high absorption; for lead, platinum and iron the indices of refraction were high, but the absorption relatively low. Very high absorption indices were obtained for stibnite and galena. Kundt discovered phenomena of double refraction of light transmitted through extremely thin electrically deposited metallic films.

the spectrum* by absorption bands. These rejected colors are reflected.

When white light is used the reflected or *surface* color is strikingly unlike the transmitted or *body* color, for instance, a drop of fuchsin solution dried on a glass plate shows the surface color green with metallic lustre and the body color red, or if a glass be laid over the surface color is blue-green. A crystal of potassium permanganate polished to a thin layer on glass gives a yellow surface color and a dark violet body color.

The surface and body colors are only approximately complementary as Wiedemann† has shown. The surface colors in uniaxial crystals, moreover, are non-polarized when reflected from basal sections, but dichroic, that is, elliptically polarized and made up of two components vibrating in and at right angles to the plane of reflection for other sections. For instance, with crystals of magnesium platinocyanide the light reflected from the basal plane is violet and not to be decomposed, whereas the green color from a prism face is always resolvable into two colors varying with the angle of incidence.

Fluorescence.

Certain organic compounds, fluorite containing dissolved organic matter and uranium glass possess the property of absorbing light and again emitting it as light of a different color. If sunlight is focussed on a dilute solution of quinine sulphate it becomes deep blue near the surface of entrance. If the spectrum of the transmitted light is examined the violet and ultra violet rays are missing, that is these have in the solution been turned into blue. A test tube of the solution held in a solar spectrum beyond the visible rays becomes blue.

In fluorite and uranium glass, which are isotropic, the color produced is independent of the direction of transmission. In the uniaxial magnesium platinocyanide crystals‡ with sunlight passed through blue or violet glass the green surface color disappears on the prism faces, and by polarizing either the incident or emerging light a dichroic fluorescence is observed, scarlet red for vibrations

*Christiansen found that the spectrum of the red solution of fuchsin lacked green and that blue was deviated more than red.

†*Ann. de Phys.* v. 151, p. 625, 1874.

‡Lommel in *Ann. de Phys.*, VIII., 634, XLIV., 311.

normal to the optic axis, orange yellow for vibrations parallel to the optic axis. If the violet rays are incident normal to the base, polarization reveals only one color, orange yellow, whatever the azimuth of the vibrations.

PHOSPHORESCENCE.

Phosphorescence, or the emission of light by a substance in the dark after it has been exposed to light, heat, friction, mechanical force, or an electrical discharge, has not been clearly shown to be dependent in any way upon the crystalline structure. It is developed by one process or another in a large number of minerals and salts.*

THE NORREMBERG AND REUSCH COMBINATIONS OF MICA PLATES.

The optical phenomena of combinations of differently oriented layers of a doubly-refracting substance are of great value in the development of theories of structure. Muscovite in which the acute bisectrix is nearly normal to the cleavage plane is especially convenient for these experiments.

If two plates of muscovite of equal thickness are laid so that their axial planes are at right angles, then with parallel light and crossed nicols, the phase difference originated in one is exactly compensated by that in the other, and the field will be dark throughout rotation of the stage. With convergent light four axes will emerge equally distant from the centre and between them thin hyperbolic color curves, the asymptotes to which form a black cross.

Norremberg discovered† that if the plates were made so thin that the phase difference of the rays developed in any plate was less than a wave-length, and that if a pile of 12 to 36 of these were made the axial planes in adjacent plates being at right angles, then not only did the compensation of the phase differences give a dark field with parallel light and crossed nicols, but with convergent light the isochromatic curves produced by the oblique rays were circles, that is, the effect of the pile was exactly equivalent to a basal section of an uniaxial crystal, Fig. 280.

Reusch found,‡ by piling 12 to 36 similar plates so that the axial planes of adjacent plates were at 120° to each other as in Fig. 281,

*Lommel in *Ann. de Phys.*, VIII., 634, XLIV., 311.

†D. Hahn, Phosphorescenz der Mineralien, *Zeit. f. d. ges. Naturwiss.*, Bd. XLIII., 1, 131, 1874.

‡ *Berichte. Ak. Wissens. Berlin*, July 8, 1869, p. 530.

that with parallel incident light there emerged a plane polarized ray, the vibration plane of which had been rotated in the direction of the piling through an angle proportionate to the thick-

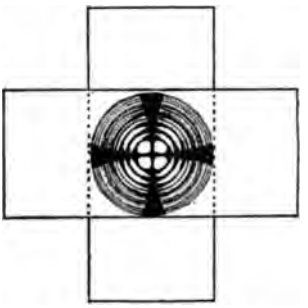

FIG. 280.

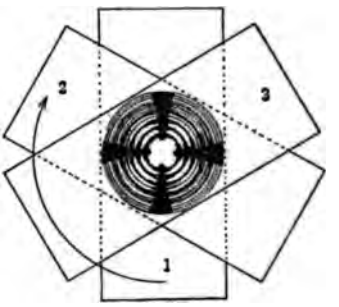

FIG. 281.

ness of the pile that is, if starting at the lowest plate the piling had been with a change in the direction of the hands of watch the rotation was also in that direction, and the color produced with white light would, by rotation of the analyzer, also in the direction of the hands of a watch, pass in the order, red, orange, yellow, green, blue, violet.

With convergent light Fig. 281 resulted, the centre being colored.

This conforms strictly to the behavior of a basal plate of right-handed quartz, except that by rotation of the stage, a slight change of color takes place, this, however, decreases as the mica plates are made thinner.

If the piling were in opposite direction the rotation of the plane of vibration would be opposite and the color order also. Based on these and similar experiments, theories as to the structure of certain crystals have been developed by Mallard,* Sohncke and others.

* *Traite de Cristallographie* II., 262–304; *Zeit. f. Kryst.*, etc., XIX., 529.

PART III, THE THERMAL, MAGNETIC AND ELECTRICAL CHARACTERS, AND THE CHARACTERS DEPENDENT UPON ELASTICITY AND COHESION.

CHAPTER XII.

THE THERMAL CHARACTERS.

Certain substances, such as halite, are essentially transparent to both heat and light; and if sunlight be decomposed by a prism of such a substance there is obtained not only the visible color spectrum consisting of the rays between the least refracted red ($\lambda = 760.4\ \mu\mu$) and the most refracted violet ($\lambda = 393.3\ \mu\mu$), but if each ray is tested by a delicate thermo-pile the temperature will be found to increase from the violet end towards the red *and to a certain distance beyond the red* where it is a maximum and thereafter to decrease, proving the existence of invisible heat rays beyond the red.

Experiments show also that these heat rays are reflected and refracted and absorbed like light rays, that they may be doubly refracted, as first shown by Knoblauch in calcite, that they may be polarized by reflection or refraction and that when polarized their power to penetrate crystals varies not only with the direction of transmission but with the direction of vibration. For instance, heat will not penetrate crossed nicols, but the interposition of a plate of doubly-refracting substance will permit components to penetrate, except at intervals of 90° when the planes of vibration of the heat rays produced in the plate coincide with those of the nicols. Interference and circular polarization of heat rays have also been experimentally proved.

Heat rays may, therefore, be regarded as invisible light rays which are in general of greater wave-length and less refrangibility than the visible rays, but are subject to the same laws. It is possible, though difficult, to determine experimentally a series of constants for crystals with respect to these invisible rays, as in the famous experiments of Knoblauch.*

* Knoblauch, *Pogg. Ann.*, LXXXV, 169; XCIII, 161.

Simpler and more convenient tests can be made with respect to the conductivity or rate of transmission of heat from particle to particle, and to expansion. For both a dependence is found to exist upon the crystal structure.

HEAT CONDUCTIVITY.

The shape of the "surface of conductivity" or isothermal surface, the radii of which are the rates of conductivity in different directions, is most easily determined from the surface conductivities in sections of known orientation.

The relative surface conductivies are most satisfactorily obtained by Röntgen's modification of the older de Senarmont method. The previously cleaned and polished section is breathed upon, quickly touched by a very hot metal point normally applied, and instantly dusted with lycopodium powder. The section is then turned upside down and tapped carefully, when the powder falls from where the moisture film had been evaporated, but adheres elsewhere, giving a sharply outlined figure. The entire operation should take less than three seconds.

In all homogeneous substances in which the surface tested is large enough to allow for irregularity of contour the resultant figure is either an ellipse or a circle, that is a section of an ellipsoid.

The major and minor axes of the ellipse are carefully measured under the microscope with a micrometer eyepiece.

In the old method of de Senarmont the surface of the section is coated with a very thin layer of wax or paraffin either applied with camels' hair brush * or by melting a thin bit on the surface and pouring off the excess. The wax-covered surface is placed in a horizontal position and heat applied at one point either:

(*a*) By a wire or narrow tube of metal (silver, platinum, copper) passing through a hole pierced normal to the waxed surface and heated either by a lamp at some distince,† or by an electric current,‡ or by a current of hot air. §

(*b*) By touching with a hot pointed wire vertically applied.

(*c*) By a little platinum ball soldered to a platinum wire, the latter being surrounded to avoid radiation and heated by an electric current.

* Tyndall, Heat as a Mode of Motion, p. 189, D. Appleton, 1891.

† de Senarmont, 1847, *Ann. de Chim. et de phys.*, XXI., 457; XXII., 179.

‡ Tyndall, l. c., 190.

§ Jannetaz, *Bull. soc. Min.*, I., 19.

Voigt recommends* a mixture of 3 parts elaidic acid with 1 part of wax, the mixture melting at 40° to 50° C.

A constant temperature is maintained until the coating has melted far enough from the point of application of the heat. The boundary of the melted patch is then the isothermal curve showing the distances which the heat has been transmitted, and is visible after cooling as a ridge which is always in the shape of an ellipse or circle.

As a result of the experiments it is found that:

1° In singly-refracting crystals (isometric) all sections yield circular isothermal curves; that is, the isothermal surface is a sphere.

2° In optically-uniaxial crystals (tetragonal or hexagonal) basal sections yield circular curves, but all other sections yield elliptical curves which become more eccentric as the section becomes more nearly parallel to the optic axis. That is, the isothermal surface is an ellipsoid of revolution around $\dot{c}$, the optic axis. As for light a division may be made into + and —. For instance, in quartz the conductivity is more rapid parallel $\dot{c}$ and in calcite at right angles to $\dot{c}$, hence quartz is thermally positive, calcite thermally negative.

3. In optically-biaxial crystals the isothermal curves are all ellipses and the isothermal surface is a triaxial ellipsoid, the axes of which are at right angles to each other. In orthorhombic crystals these axes coincide with the crystallographic axes, and the major and minor axes of the ellipse obtained in any principal section are parallel to the crystal axes therein. In monoclinic crystals one axis of the isothermal surface coincides with $\dot{b}$ and the other two lie in the plane 010. In the zone [010 100] the major and minor axes of the isothermal curves should coincide with crystal axes. In triclinic crystals there is no essential or probable parallelism of crystal axes and axes of ellipses in any section.†

* *Elemente der Krystallphysik*, 78.

† Jannetaz showed an apparent connection between directions of cleavage and ease of conductivity. For instance, the isothermal surface for uniaxial crystals which have an easy basal cleavage is usually a flat oblate spheroid. If the direction of cleavage is not basal, but oblique, the longer axis will often be in the section most nearly parallel to the cleavage. As will be later explained, planes of cleavage are supposed to be directions of greatest closeness of molecules, and it seems, therefore, that the rapidity of transmission increases as the molecules are closer together. It is curious, however, that if an amorphous substance, such as glass, is subjected to compression or tension the isothermal circles become elliptical, the shorter axis being in the line of greater pressure or packing of molecules.

EXPANSION BY HEAT.

Most * substances increase in volume when heated and contract on cooling, and experiments show that in crystals the rate of expansion is not the same for directions crystallographically unlike, but varies as follows:

1° *Isometric Crystals.* The rate of expansion is the same for all directions. A sphere heated becomes a sphere of larger diameter.

2° *Tetragonal and Hexagonal Crystals.* The rate of expansion is the same for all directions equally inclined to the axis c, is either a maximum or a minimum parallel to c, and varies regularly from this to the direction at right angles. A sphere heated becomes an ellipsoid of revolution around c as an axis.

3° *Orthorhombic, Monoclinic and Triclinic Crystals.* There is no axis of isotropy. The directions of maximum and minimum expansion are at right angles, and a sphere heated becomes a triaxial ellipsoid, the axes being these directions and a third at right angles to their plane.

Orthorhombic, the axes coincide with the crystal axes.

Monoclinic, one axis coincides with b, the others lie in 010, but not coincident with conductivity axes or principal vibration directions.

Triclinic, the axes have no fixed positions relative to the crystal axes.

Direct Measurement of Linear Expansion.

The coefficient of linear expansion, that is the increase in a unit of length for a temperature change from 0° to 1° C. may be accurately measured for any direction by the method of Fizeau † which as perfected by Abbe is as follows: ‡

Fig. 282.

In Fig. 282 O is a plane parallel plate of the crystal, about 10 mm. thick, which rests upon three projections turned upon the steel plate T. Three screws, with a fine 2 mm. thread, pass through T and support the glass plate P, which tapers slightly so that

*A few expand on cooling below a certain point and one, at least, iodide of silver, contracts when heated above the ordinary room temperatures.

† Description by J. R. Benoit, *Trans. et Mem. du bur. internat. des poids et mes.*, I., 1, 1881; VI., 1. 1888.

‡ Described by C. Pulfrich, *Zeit. f. Instrumentkunde*, XIII., 365, 401, 437.

when the upper surface is horizontal the lower is inclined about 20 minutes. By adjusting the screws a thin wedge-like film of air is left between the horizontal polished surface of the substance and the lower surface of the glass.

The telescope *PO*, Fig. 283, is at once telescope and collimator, that is the light from a Geissler tube, shown at *L* in the smaller

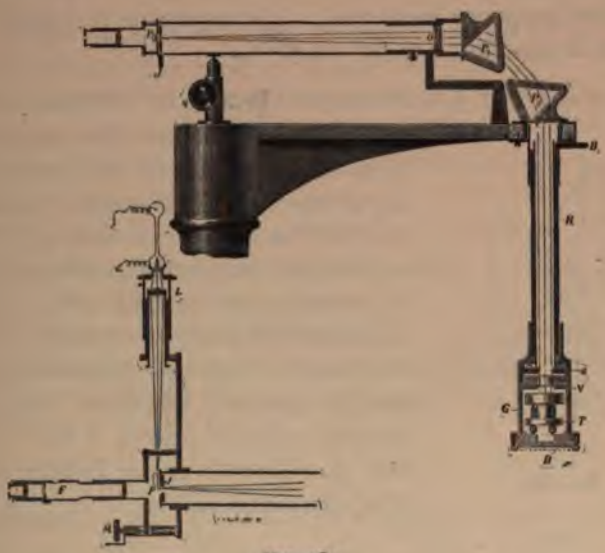

FIG. 283.

figure, enters at the side, is deflected by the prism *P*, made parallel by the lens system *O*, decomposed by the two flint glass prisms P_1 and P_2 and by varying the angles at which these are set by the screw *S*, rays of any chosen color are made to pass vertically through *R* and reach the interference apparatus within the brass vessel *G*.

The rays incident at the upper surface of the crystal plate and the lower surface of the glass plate interfere on reflection producing parallel dark bands wherever the thickness of the air film is $\frac{1}{2}\lambda$, $\frac{3}{2}\lambda$, etc. The distance between two adjacent bands is therefore a function of the wave-length of the light used. This distance is measured by a screw micrometer *M* which moves a vertical double hair horizontally across the field.

The vessel *G* containing the interference apparatus is enclosed in two concentric cylinders, the outer one containing a liquid. When the liquid is heated the crystal plate and the interference

apparatus both expand and the interference bands change in distance apart. From this change and from the previously determined effect of the expansion of the apparatus the change due to the expansion of the crystal is calculated.*

Voigt describes† a simpler apparatus in which the expansion of the crystal tips one of two parallel mirrors. Reflected signals from the two mirrors are viewed by a telescope one meter distance and their divergence measured. The apparatus is heated by immersion in paraffin oil at 60° to 70° C.

MEASUREMENT OF EXPANSION BY CHANGE IN DIEDRAL ANGLES.

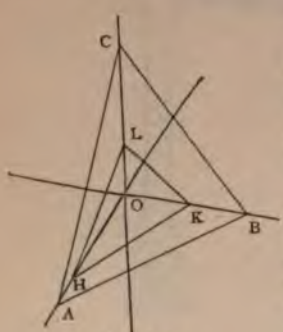

FIG. 284.

The indices and therefore the zone relations are not changed by uniformly heating a crystal, for any series of points on a straight line remains on a straight line and at the same proportionate distances apart. If, therefore, in Fig. 284 *ABC* is the unit plane and *HKL* any other plane, the points *O*, *K* and *B* will after expansion be on a straight line and the same proportionate distance apart, so also *O*, *L* and *C* or *O*, *H* and *A*. Therefore, the indices of *HKL* after expansion will be

$$\frac{OA'}{OH'} = \frac{OA}{OH} = h, \quad \frac{OK'}{OB'} = \frac{OK}{OB} = k; \quad \frac{OL'}{OC'} = \frac{OL}{OC} = l; \text{ or } hkl.$$

In *isometric crystals* the angles are unchanged by expansion, but in all other systems all diedral angles are altered except that:

(*a*) Faces parallel to two expansion axes remain parallel to their original position and normal to all faces in the zone of the third axis.

(*b*) If the rates of expansion are equal parallel to two axes the faces in the zone of the third axis remain parallel to their original positions.

In *tetragonal and hexagonal crystals* by (*a*) the basal planes remain at 90° to the faces in the prismatic zone, so also any face in the prism zone remains at 90° to all faces of the zone originally normal to it and by (*b*) all angles in the prismatic zone remain constant.

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* *Zeit. f. Instk.*, XIII., 440, etc.

† *Wied. Ann.*, XLIII., 831, 1891.

In *orthorhombic* crystals since a, b and c are expansion axes, each pinacoid remains at 90° to every face in the zone of the third axis.

In monoclinic crystals the clino pinacoid remains at 90° to faces in the zone of b.

From the change in angle the linear coefficients may be calculated.*

The change in angle is usually small, requiring delicate measurement. It may be demonstrated† by cutting a plate-like cleavage of calcite, Fig. 285 *a*, normally and parallel to the longer diagonal of the larger face, reversing one-half and gluing as in *b* with water glass. On heating, the adjacent angles expand in the same direction and, as exaggerated in *c*, one-half becomes inclined to the other about 20′ for 100° C. temperature change. This bending is easily shown by reflection of a signal on a screen 3 meters distant.

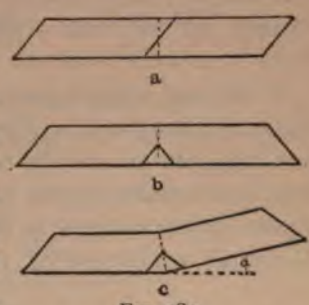

FIG. 285.

DETERMINATION OF EXPANSION BY CHANGES IN THE OPTICAL PHENOMENA.

The expansion of a crystal produces a change in the rapidity of transmission of light which may be determined by measurement of the indices of refraction or indirectly by the interference phenomena.

In *singly refracting* (isometric) crystals the change is alike in all directions. The crystal remains optically isotropic but the index of refraction may become smaller as with fluorite, or larger, as with diamond.

In *uniaxial* (tetragonal and hexagonal) crystals the changes in velocity are unequal, that is the principal indices of refraction and

* For instance if the rhombohedral angle of a calcite cleavage, which decreases almost 9′ for 100° elevation, is 105° 5′ at 10° C. and 104° 56′ at 110° C. then by formulæ, $\sin a = \cos \frac{1}{2} X \times 1.155$, $\tan a \times .866 = c$, (Moses' and Parson's Mineralogy, p. 60) we have $l \sin a = 9.84663$, $a = 44°.63$, $\tan a \times .866 = c = .8549$, and $l \sin a' = 9.84732$, $a' = 44° 72$, $\tan a \times .866 = c' = .8575$.

† Voigt, *Elemente der Krystalphysik*, p. 49.

a vary unequally and not necessarily in the same direction.* The strength of the double refraction $\gamma - a$ may be either increased, raising the interference color and contracting the rings in the interference figure, or the reverse may take place. The change when $\gamma - a$ is made less may, for a certain temperature, reduce the double refraction to zero, that is *for that color of light and at that temperature* the crystal is optically isotropic and for a further temperature change in the same direction will change in optical character from + to − or − to +.

In *biaxial* crystals (orthorhombic, monoclinic and triclinic) the unequal changes in a, β and γ, the principal indices of refraction, are indicated not only by raising or lowering the interference colors and in the contraction or expansion of the rings of the interference figure but by changes in the angle between the optic axes, which is simply a function of the principal indices. If either a or γ at any temperature become equal to β then for that temperature and light the crystal is uniaxial and the axial angle is zero and for a further change in the same direction the former middle index β will become a or γ, that is $\mathfrak{b}$ will become $\mathfrak{a}$ or $\mathfrak{c}$ and the optic axes will pass into a plane at right angles to the former position of the axial plane. Fig. 286 represents successive changes in such a transformation.

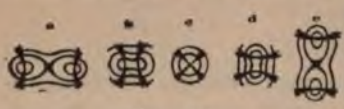

FIG. 286.

The following selected examples illustrate these facts.

BARITE (Orthorhombic). $2E$ for sodium flame *increases* with rising temperature from 64° 1′ at 20° C. to 68° 51′ at 100° C. and 77° 16′ at 200° C.

GLAUBERITE (Monoclinic). The principal vibration directions are essentially unchanged, but $2E$ *decreases* with rising temperature. Within a range of 40° C. the crystals become successively uniaxial for all colors and the axes thereafter are in a plane at right angles to the former axial plane.

Temperature Centigrade.	Apparent Angle $2E$ for Li Red.	Na Yellow.	Tl Green.	Blue.
18°	———	———	———	0°
22°	13° 30′	11° 8′	8° 14′	———
36°	11° 1′	8° 9′	0°	8° 42′
46°	8° 40′	0°	7° 8′	11° 8′
58°	0°	7° 14′	10° 32′	13° 2

* With increased temperature:

	a	γ	$\gamma - a$
Calcite	increased	increased	increased
Beryl	increased	increased	decreased
quartz	decreased	decreased	decreased

GYPSUM (Monoclinic). The angle $2E$ decreases with rising temperature and the principal vibration directions change, the dispersion changing from inclined to horizontal.

Temperature. Centigrade.	$2E$ for Na Yellow.	Temperature Centigrade.	$2E$ for Li Red.
20°	92°	20°	96°
50°	79°	47°	76°
100°	51°	71°	59°
116°	36°	95°	39°
134°	0°	116°	0°

The test is usually made by replacing the glass vessel *M*, Fig. 273, by a metal air bath, Fig. 287, consisting of a rectangular hol-

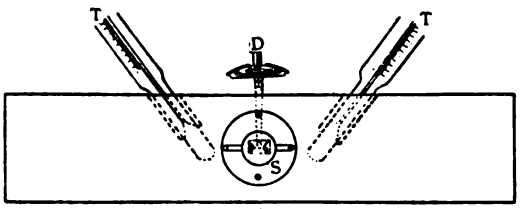

FIG. 287.

low box of copper, which projects on either side beyond the polariscope, and is heated by gas burners. At the top is an opening for the crystal carrier, and at the large sides are glass windows so set in tubes, that by a key the distance apart can be made as small as the crystals will permit. From the top of the box project two thermometers *T* reading to 300° C.

The crystal section is adjusted, and the angle is determined at room temperature, the two burners are then lighted and the heating continued until a constant temperature can be maintained for one-half hour, when a new reading is made; this is repeated for different intensities of flame.

CHAPTER XIII.

THE MAGNETIC AND ELECTRICAL CHARACTERS OF CRYSTALS.

MAGNETIC INDUCTION OF CRYSTALS.

All substances are either attracted or repelled in some degree when in the field of a strong electromagnet. If attracted they are said to be "*paramagnetic*" or "*magnetic*;" if repelled they are "*diamagnetic*."

If a rod of any substance is suspended by a fibre so as to swing freely horizontally between the vertical poles of an electromagnet, *ab*, Fig. 288, magnetic induction takes place and as the lines of force between the poles are essentially horizontal, the effect of the pull or thrust upon rotation is greatest for the particles furthest from the axis of rotation. If paramagnetic, therefore, the effect is to pull the rod into a longitudinal or "*axial*" position with its ends as near the poles of the magnet as possible, and if diamagnetic the rod is pushed into a transverse or "*equatorial*" position with its ends as far from the magnetic poles as possible.

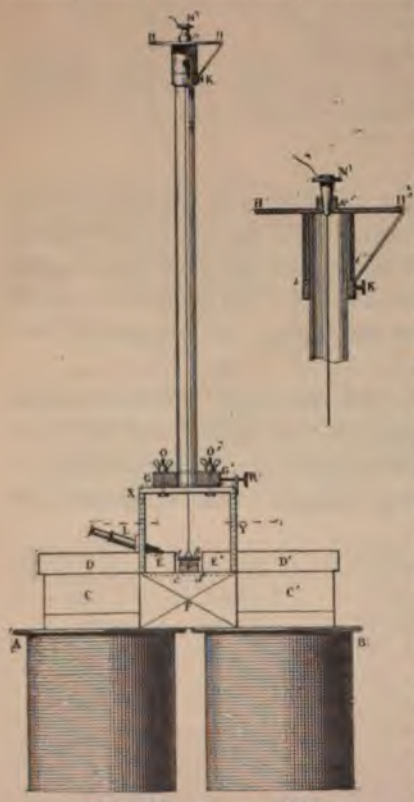

FIG. 288. $\frac{1}{10}$ natural size.

Fig. 288 shows the apparatus of Edmond Becquerel* in which *A* and *B* are the poles of a large electro-magnet, *C* and *C'* square soft iron pole pieces and *DE*, *D'E'* long narrow soft iron pole pieces placed so that

* *Ann. de Chim. et de Phys.*, 1850, V., 28, p. 283.

the back face of *D E* and front face of *D' E'* lie in the same plane through the torsion thread. Bars of 25 mm. long, 2 to5 mm., broad are suspended in the position *a b* and adjusted by the torsion head *N* until a scratch at one end of the bar coincides with the cross hair of the telescope *L*. The current is then turned on and the deviation due to attraction or repulsion observed and measured by the amount it is necessary to turn the circle *H H'* to restore the original position.

With crystals, however, the particles in certain directions become more strongly magnetized than in others, and the para- or diamagnetism is judged by hanging a thin glass tube, filled with powder of the substance, between the magnetic poles, the particles of the powder having all possible orientations all effect of direction is eliminated.

To Determine the Relative Strength of Magnetization in Different Directions in a Crystal.

The crystal is suspended by a silk fibre and should be of such a shape that the section normal to the axis of suspension is circular. If, however, the crystal is small, the form is less important.

Plücker used for his experiments a large electromagnet with six Groves elements, the poles of the magnet being 1.6 inches apart and the space around the poles protected from currents of air by a glass cage. The crystal was suspended by a double silk thread from a torsion balance.

The direction, in the section normal to the axis of suspension, which is most strongly affected will evidently take an *axial* position with paramagnetic crystals and an *equatorial* position with diamagnetic crystals.

By suspending a cube by its three rectangular axes, *a*, *b* and *c*, successively the magnetic intensities in these directions may be compared, for example:

	Suspended by:		
	a	*b*	*c*
Axial direction assumed by	*b*	*a*	*b*
Equatorial direction assumed by	*c*	*c*	*a*

If the crystal is diamagnetic, then, since *c* was twice equatorial, *c* is the axis of greatest magnetization, but if paramagnetic, then is *b* the axis of greatest magnetization.

The relations cannot yet be said to be well understood as very

few minerals have been thoroughly tested. It does not appear that isometric crystals are magnetically isotropic, for the latest investigations of magnetite show that prisms cut parallel to a ternary axis are most strongly magnetized, those parallel to a binary axis only a little more feebly and those parallel to a quaternary axis much more feebly.*

The experiments of Plücker † appear to prove that all crystals of other systems are magnetically anisotropic and that a suspended sphere in a uniform magnetic field is in stable equilibrium only when the direction most strongly magnetized is "axial," that is, is parallel to the lines of force.

In hexagonal and tetragonal crystals the direction of maximum magnetization is either parallel or at right angles to the vertical axis *c* and the crystal is said to be magnetically positive or magnetically negative. If a sphere is suspended with *c* horizontal then four cases ‡ result.

Paramagnetic positive, the position of *c* is axial.

Paramagnetic negative, the position of *c* is equatorial.

Diamagnetic positive, the position of *c* is equatorial.

Diamagnetic negative, the position of *c* is axial.

In orthorhombic, monoclinic and triclinic crystals the directions of maximum and minimum intensity, are at right angles to each other and the intensity of the third axis of the "induction ellipsoid" is at right angles to their plane.

In orthorhombic crystals § these axes are the crystal axes *a*, *b*, *c*. In monoclinic crystal one axis is parallel to *b*, the others are in 010, but not necessarily or probably parallel either to axes of

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* Aimantation non isotrope de la Magnétite cristallisée.—P. Weiss.—*C. R.*, CXXII. 1405.

† *Pogg. Ann.*, v. 72, p. 315; v. 76, p. 576; v. 77, p. 447; v. 78, p. 427; v. 86, p. 1.

‡ Examples are:

+Paramagnetic,	—Paramagnetic,	+Diamagnetic,	—Diamagnetic,
Siderite,	Tourmaline,	Calcite,	Bismuth,
Wernerite,	Beryl,	Mimetite,	Arsenic,
Torbernite,	Vesuvianite,	Wulfenite,	Zircon.

§ Examples are:

		Strength of Magnetization.		
		$\varphi_1 >$	$\varphi_2 >$	φ_3
Topaz,	Paramagnetic,	*a*	*c*	*b*
Anhydrite,	Diamagnetic,	*a*	*b*	*c*
Barite,	Diamagnetic,	*c*	*a*	*b*
Epsomite,	Diamagnetic,	*c*	*b*	*a*

thermal conductivity or optical principal vibration directions. In triclinic crystals there are no fixed relations.

Practically no satisfactory determinations have been made of absolute values of coefficients.

TRANSMISSION OF ELECTRIC RAYS.*

Electric waves differ from light waves only in their vastly greater length, they travel with the same velocity and exhibit similar phenomena. Many substances opaque to light waves are transparent to electric waves, and in this lies the hope of a series of tests for optically opaque crystals corresponding to the series for optically transparent crystals.

Professor Bose† describes an apparatus, essentially an electric polariscope, the polarizer and analyzer consisting of wire gratings, made by winding fine copper wire 2 mm. diameter around a thin sheet of mica (about 25 lines per cm.); the mica pieces are then dipped in melted paraffine, after which a round disc is cut from the sheet. The electric waves are produced by a small Ruhmkorff coil causing oscillatory discharges between two small (1½ cm.) metallic spheres; beyond these in the same tube is a convex lens with a spark gap at its principal focus, then follow in order the grating polarizer, the crystal, the grating analyzer, a modified coherer, and a connected, distant d'Arsonval galvanometer.

When the gratings are crossed no current is shown, the introduction of the crystal produces a current which is shown by the throw of the needle reflected by a mirror upon a scale. When the principal vibration directions of the crystal coincide with those of the gratings no current passes. Crystals of moderate size are successfully tested by this apparatus.

ELECTRICAL CONDUCTIVITY.

Electrical conductivity, although varying between very wide limits in different substances,† appears to be dependent much more

* The Polarization of Electric Rays by doubly refracting Crystals.—J. C. Bose, *Jour. Asiatic Soc.*, Bengal, LXIV., 291, 1895.

† No substance possesses absolute electrical resistance; practically, however, conductivity may be considered to be limited to the metals; some metalloids; most sulphides, tellurides, selenides, bismuthides, arsenides and antimonides, some of the oxides; and, at higher temperature, a few haloids.

upon the constitution of the chemical molecule than upon the crystalline structure. A certain dependence upon crystallographic direction has, nevertheless, been observed in a few substances.

The principal experiments are those of Wortman,* and the more recent series by Beijerinck.† Both used essentially the same method, in which a prism of known dimensions was introduced into a direct weak current, the strength of which was varied by resistances and the deviation observed in a galvanometer.

Good contact was obtained by using tough copper amalgam or sometimes bright sheet lead or simply graphite rubbed on with a lead pencil. Natural faces were cleaned with acid, caustic soda, water and alcohol, and sometimes even with hydrofluoric acid.

To study the effect of temperature the substance was heated in a small air bath made with double walls of asbestos and covered with copper, thus giving a very regular distribution of temperature without thermo-electric effects.

The principal results bearing upon crystal structure are as follows:

Isometric Crystals.

Magnetite, the electrical conductivity is essentially alike in all directions.

Tetragonal and Hexagonal Crystals.

Hematite,‡ hexagonal, the electrical conductivity parallel to c is essentially twice that normal to c and for any direction making the angle α with c; the resistance is $\omega = \omega_a \sin^2\alpha + \omega_c \cos^2\alpha$ in which ω_a and ω_c are the resistances normal and parallel to c.

Cassiterite, tetragonal, and zincite, hexagonal, give conformable results.

Orthorhombic Crystals.

Marcasite and bismuthinite, show the greatest and least resistances parallel to two of the axes a, b, c.

That is, the few tests recorded show that the electrical conductivity of crystals conforms to the thermal conductivity.

THERMOELECTRIC CURRENTS.

If a metallic circuit is made by soldering together one end of each of two rods of different metals and connecting the other ends

* Mem. de la Soc. d. hist. Nat. de Genève, XIII., 1853.

† Ueber das Leitungsvermögen der Mineralien für Elektricität.—F. Beijerinck.—*Neues Jahrb. Min.*, Beil. Bd., XI., 403, 1896–7.

‡ H. Bäckström.—*Öfv. d. K. Vetenskaps-Ak. Förh.*, 1894, No. 10, 545.

by wire, heating or cooling the junction will develop an electric current the strength of which will depend upon the change of temperature and upon the metals used.

In a crystal the electromotive force of the current in part depends upon the crystallographic direction, and a thermoelectric current may be produced:

(*a*) By coupling rods cut in different directions from the same crystal and heating the junction.

(*b*) By inserting a rectangular parallelpipedon longitudinally in a metallic circuit the sides being kept at a constant equal temperature but the ends differing. For example, Bäckström* placed hematite between two sheet-copper boxes, through one of which water was flowing and through the other steam. The boxes were connected by wires with a Lippman capillary electrometer.

(*c*) The end surfaces may be held at the same temperature† and two opposite side faces at different temperatures.

Friedel‡ clamped three plane parallel plates between two platinum wires, touching equal spaces on the crystal, connected the plates with a galvanometer and placed them in a space uniformly heated by a water bath.

Slight impurities appear to produce very notable changes in the thermoelectric results.

DIELECTRIC INDUCTION IN CRYSTALS.

A crystal suspended in an electrostatic field develops in an almost inconceivably short period§ an electric polarity, and the crystal tends to assume a position in which the lines of force and the direction of maximum induction are parallel.

According to the experiments of Root‖ the directions of maximum and minimum induction are respectively the principal vibration directions c and a in light transmission. In these experiments circular plates of tourmaline, quartz, and calcite, about 10–11 mm. in diameter and cut parallel to the optic axis, were attached by a little drop of glue to a silk thread and suspended between the vertical plates of a condenser charged by a rapidly alternating

* Last cit.

† Liebisch, *Grundriss der Phys. Kryst.*, 217.

‡ *Ann. de Chim. et de Phys.*, 1869, XVII., 79.

§ Less than 0.0000821 second, according to Root.

‖ *Poggendorf's Annalen*, CLVIII., 1, 425, 1876.

current. The vibration through an arc of 10–20 minutes thereby produced in the plate was reflected to a telescope two meters distant, and its period determined both when the direction of maximum induction was vertical and when horizontal. The quotient of the former by the latter gave a basis of comparison. Aragonite and topaz were also tested. For instance,

	Period of vibration when direction of maximum induction is		Quotient of $\frac{A}{B}$
	A. Vertical.	*B*. Horizontal.	
Aragonite	4.1125	3.768	1.091
"	4.0500	3.668	1.105
Calcite	2.240	2.175	1.022

The condenser Fig. 289 consisted of a ring *C* of gutta percha serving as a frame for the two vertical brass plates *A* and *B*. Through *C* passed the glass tube *D* with a torsion head to which the discs were hung between *A* and *B* by a single thread of silk. The motions of the disc were observed through the opposite glass windows *E*. A voltaic current passing through a commutator produced alternately + and − charges in *A* and *B* in some instances as rapidly as 6,000 alternations per second.

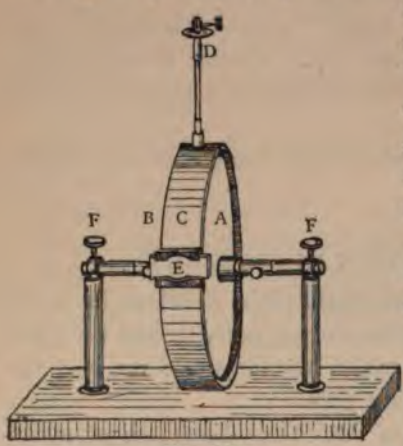

FIG. 289.

As all dielectrics have some degree of conductivity the electric polarity is apt to be modified by the more slowly developed conductivity phenomena.

The effect of conductivity in use of the constant current was shown by suspending a calcite plate with *c* horizontal. *c* was at first axial but in less than two minutes turned reversing the poles, whereas with alternating current *c* was equatorial.

The strength of the induction in any direction is indicated by the so-called dielectric constant* which may be determined by

* Maxwell assumes to be proportionate to the square of the constant *A* in Cauchy's formula for dispersion, $n = A + \frac{B}{\lambda^2}$ or, since if $\lambda = \infty$ $n = A$, to the square of the index of refraction for light of infinite wave-length vibrating parallel the given direction. (Groth, *Phys. Kryst.*, III. Ed., 194.)

the method of Bolzmann * in which the attractions exerted are measured as follows: A small sphere of the dielectric to be tested is attracted by a fixed charged metal sphere and the amount of its deviation h determined by the deflection of a mirror; the sphere is then replaced by another sphere of the same diameter but coated with tin foil and the deviation h' of this produced by the same charge in the metal sphere is determined; then, according to Bolzmann, if ε is the dielectric constant

$$\varepsilon = \frac{\frac{h'}{h} + 2}{\frac{h'}{h} - 1}$$

either for uniaxial crystals or for biaxial crystals in which an axis of dielectric symmetry is parallel to the lines of force.

The constants may also be determined by comparison of capacity.†

From the dielectric constants in the principal directions the constants for any direction may be calculated.‡

If $D_a D_b D_c$ are the principal constants the constant D_r for any direction is:

$$D_a \cos^2(a.r) + D \cos^2(b.r) + D_c \cos^2(c.r)$$

The conductivity on the plane surface of a dielectric crystal theoretically must conform to the dielectric constants in different directions.

Wiedemann's§ *Experiment.*—An insulated needle was fastened in proper holder in normal contact to a crystal face, previously covered with a poorly conducting powder such as lycopodium or minium, and positively charged by contact with the knob of a leyden jar. The powder near the point is tossed aside in shapes which are elliptical or circular according to surface. The results can be made permanent by collodion. In general the longest axis is in the vibration direction of the light of greatest velocity.

de Senarmont's‖ *Experiments.*—de Senarmont coated the crystal with tin foil except that a circular hole was cut in the foil covering the face to be studied. The foil was grounded the crystal was placed in partial vacuum opposite a point of brass wire from which *positive* electricity was discharged following a direction assumed therefore to be that of easiest conductivity. The tests are made in the dark.

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* *Ber. Ak. Wien.* (2), LXX., 342, 1874, also Ch. Borel, *Arch. Soc. Phys. et nat. de Genève* (3), XXX., 131, 1893.

† Liebisch, *Grundriss der Phys. Kryst.*, 230.

‡ A. Lampa, *Ber. Ak. Wein.*, CIV., 1179.

§ *Pogg. Ann.*, 76, 404, 1849.

‖ de Senarmont, *Ann. de Chim. et de Phys.* (3), 28, 257, 1850.

PYRO-ELECTRICITY.

Equal positive and negative charges of electricity are developed at different points or poles of certain crystals during a uniform change of temperature.

A temperature change of at least 70° to 80° C. is desirable. Usually the crystal is heated in an air bath to a uniform temperature, then drawn quickly once or twice through an alcohol flame to remove any electricity occurring on the surface, and then brought into a cooler place and allowed to cool.

If heating injures the crystal it may be cooled from the room temperature by a freezing mixture.*

During the cooling of the crystal the positive charges collect at the so-called *antilogue* poles, and the negative charges at the *analogue* poles,† and may be distinguished by their effect on other electrified bodies. For instance, a cat's hair rubbed between the fingers becomes positively electrified and is attracted by the analogous pole and repelled by the antilogous pole, or as in the method of Hankel,‡ the poles may be touched by a platinum wire carefully insulated and worked by a system of levers, and the charge conducted to an especially constructed gold leaf electrometer.

Kundt's§ method is, however, most generally employed and consists in blowing upon the cooling crystal a fine well dried‖ mixture of equal parts of powdered sulphur and red oxide of lead. The nozzle of the bellows is covered by a muslin net and, in passing through, the sulphur is negatively electrified and is attracted by the positive poles coloring them yellow, while the minium is positively electrified and is caught by the negative poles coloring them red. By pressing the crystal upon sticky paper a permanent record can be obtained. The dust should fall evenly and the bellows be held far enough away to prevent direct action of the blast.

The figures here shown¶ represent crystals dusted with sulphur and minium during cooling, the darker dotted portions showing the analogue poles, the hatched portions, the antilogue poles.

* Snow or ground ice and salt; 3 pts. snow or ground ice with 1 pt. H_2SO_4; 2 pts. snow or ground ice with 3 pts. crystals $CaCl_2$.

† With rising temperature these are reversed.

‡ G. W. Hankel, Inaug. Dissertation.

§ *Pogg. Ann.*, CXXXVI., 612, 1862; *Weid. Ann.*, XX., 592, 1883; XXV., 145. 1886.

‖ Dry over H_2SO_4 in a vessel from which the air has been partially exhausted.

¶ Drawn from the colored plate III. in Groth, *Phys. Kryst.*, III. ed.

Fig. 290 represents a tourmaline crystal, Fig. 291 a calamine crystal, in both of which polarity is shown with reference to the

FIG. 290.

FIG. 291.

FIG. 292.

vertical axis. Fig. 292 represents a boracite crystal, showing polarity with reference to four axes, and Fig. 293 a quartz crystal with three axes of polarity. Fig. 294 shows a basal section of quartz.

FIG. 293.

FIG. 294.

FIG. 295.

Quantitative examinations were made by Gaugain* by coating the ends of a cylinder of the crystal with tin foil, connecting one end to the ground and the other to some form of self-discharging electroscope. The number of discharges are counted and serve as a relative measure especially if the capacity of the electroscope is small. In this way it was shown that with tourmaline the amount is independent of the time of cooling, is alike for the same change of temperature in either direction, and is independent of the length, but proportionate to the cross section and to the difference between the end temperatures.

THEORY OF LORD KELVIN.

Lord Kelvin theorized† that tourmaline, in which pyroelectricity was first observed, is internally in a state of uniform electric polarity, and that, therefore, at the surface there should be an

* *Ann. de Chim. et de Phys.*, III., 57.

† Math. phys. papers Sir. William Thomson, I., 315.

electric charge of uniform surface density. In a medium not perfectly insulating such as damp air there is gradually formed by induction an electric layer completely neutralizing the charge in the tourmaline, that is making it essentially non-electric.

But if the tourmaline is heated or cooled the strength of the internal polarization is altered, while that of the electrified layer of outer medium is more slowly altered and the effect is electric polarity.

Riecke * confirmed this theory by showing that, in a fairly perfectly insulating medium, tourmaline remained electrified twenty to thirty hours, *after cooling* to its normal temperature. The heated tourmaline while still non-electrified was placed under an air pump over a gold leaf electroscope and the air slightly rarefied.

PIEZO-ELECTRICITY.

Electric charges may be developed by pressure, for instance, calcite pressed between the fingers becomes positively electrified, tourmaline compressed in the direction of *c* shows a positive charge at the antilogue end and a negative charge at the analogue end or precisely the charge which would result from cooling; whereas, if the pressure is removed and the crystal allowed to expand the charges are reversed conforming to rising temperature. The quantity developed is always proportionate to the pressure.

If strains are developed by unequal heating electrical charges may be developed as, for instance, by standing a basal section of quartz upon a hot centrally placed metal cylinder, the distribution of charges conforming to the effect of cooling a complete crystal, whereas, with a hot metal ring the charges are reversed.

The methods for detecting the charges are the same as for pyroelectricity. Fig. 295 shows the distribution of the charges in a basal section of quartz produced by pressure in the direction of the arrows which conforms evidently to Fig. 294, or the effect of cooling.

For quantitative examination J. and P. Curie used the following method.†

The crystal is cut in the form of a rectangular parallelopipedon, Fig. 296, and two opposite faces, *A*, *B*, covered with tin foil. One of these, *A*, is grounded; the other, *B*, is connected with one of the plates of a condenser *C* of known capacity, and also with one

* E. Riecke, *Wied. Ann. 28*, 43, *31*, 889, *40*, 264, *49*, 421.

† Compte Rendu, XCI., 294, 383; XCII., 350.

of the couples SS of a Thomson-Mascart electrometer. The other plate of the condenser is grounded, and the couple $S'S'$ of the electrometer is connected with one pole of a Daniell cell, the other pole being grounded.

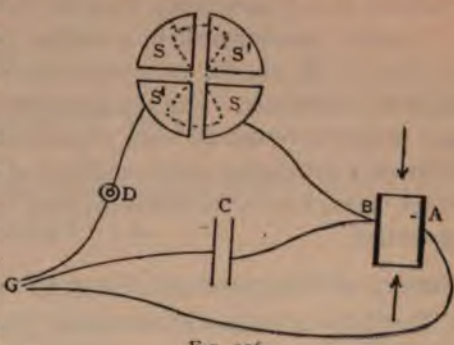

FIG. 296.

The relation between the pressure employed and the electricity developed is determined as follows: Let D denote the potential of the Daniell cell, C the capacity of the condenser, and c the capacity of the system, consisting of the plate B, the couple SS and the conductors.

For some pressure P, in the direction of the arrows, the needle of the electrometer will be at zero and the entire system charged to a potential D. That is, the pressure will have developed* a quantity $Q = (C + c)\,D$.

If the condenser is removed the pressure necessary to bring the needle again to the zero position will be P', and the quantity developed will be $Q' = cD$, hence for the pressure $P - P'$ the quantity developed is $Q - Q' = CD$, that is a quantity sufficient to charge a condenser of known capacity C to a known potential D.

THEORY OF PYRO- AND PIEZO-ELECTRICITY.†

Whenever the volume of a crystal is altered either by a temperature change or by mechanical pressure a portion of the heat energy or mechanical energy may be converted into electric energy, which, in poorly conducting crystals, will frequently be manifested by the accumulation of positive and negative charges at different points or poles.

Conversely, as shown by Lippmann,‡ a section which becomes for instance, positively electric by normal pressure must, if charged with positive electricity, *expand* in the direction of the normal. In other words, the electric charges are functions of the change in volume.

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* Quantity equals capacity times potential.

† W. Voigt, *Wied. Ann.*, LV., 701, 1895.

‡ G. Lippman, *Ann. Chim. et Phys.*, (5) XXIV, 145.

J. and P. Curie devised* a delicate apparatus for measuring the force of such expansions and the measured results for quartz, checked almost absolutely with the calculated results.

Crystals symmetrical to a central point (Groups 2, 5, 8, 13, 15, 21, 22, 25, 27, 29, 30, 32) cannot develop opposite charges at the ends of a diameter. The apparent exceptions obtained by Hankel and others have not all been explained, but may in part be due to influences dependent on the method of conducting the operation, in other cases, barite, for instance, they have been ascribed† to twin structure.

EFFECT OF ELECTROSTATIC FIELD UPON OPTICAL CHARACTERS.

Kundt‡ coated two of the sides, parallel *c*, of a quartz parallelopiped, with tin foil, connected these with the electrodes of a Holz machine and found that the circular interference rings became elliptical, the minor axes of the ellipses being parallel to the direction of expansion produced by the charge. Pockels§ shows that the change is too great to be simply a consequence of the expansion and must be in part due to a direct influence of the electric force on the light motions.

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* Brief description Liebisch, *Grundriss der Phys. Kryst.*, 481.

† Beckenkamp, *Zeit. f. Kryst.*, XXVII., 85.

‡ *Wied. Ann.*, XVIII., 228, 1893.

§ F. Pockels, *Abh. d. k. Ges. a. Göttingen*, XXXIX., 1–204.

CHAPTER XIV.

ELASTIC AND PERMANENT DEFORMATION OF CRYSTALS.

In an elastic substance the distance apart and relative position of the particles may be changed by mechanical force, and up to a certain so-called "elastic limit" the particles will, on removal of the force, regain their former position. Such a change is called an *elastic* deformation or form-alteration.

Any strain or pressure in excess of the so-called "elastic limit" produces a *permanent* change of form.

HOMOGENEOUS ELASTIC DEFORMATION.

The only recorded experiments by pressure on all sides were made upon halite by aid of a piezometer.* The cubic compression coefficients have, however, been calculated by Voigt for a number of species. The same five classes result as with the homogeneous deformation produced by changes of temperature.

The alteration produced by a change of temperature cannot be exactly compensated by uniform pressure on all sides except in the case of isometric crystals.†

ELASTIC DEFORMATION DUE TO PRESSURE IN ONE DIRECTION.

Observation shows that the extension or compression of any rod of length l, breadth b and thickness t, produced by a weight W acting in the direction of the length, is $\frac{Wl}{bt} E$, in which E is a characteristic factor called the coefficient of extension. Obviously if W, l, b and t are unity, the extension becomes equal E, that is the coefficient of extension is the extension produced upon a unit rod by a unit weight.

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* Röntgen and Schneider, *Wied. Ann.*, XXXI., 1000, 1887.

† Liebisch, *Phys. Kryst.* 1891, 576.

The most convenient method of obtaining the extension coefficient for any direction in a crystal is by cutting a thin rod R, Fig. 297, of rectangular cross section, in the given direction, supporting it, as shown, upon the two rests A and B, and loading by a weight W connected with a centrally placed knife edge. The deflection of the central section produced by the weight is determined by Koch as follows:* A rectangular glass prism P is placed just below and with one face parallel to the rod, and another face vertical. Monochromatic light is reflected upon the hypothenuse face, and, very much as in the Fizeau apparatus, p. 167, interference bands are produced by the rays reflected from the lower surface of the rod R and the upper surface of the prism P, and are viewed by a horizontal telescope through the vertical face.

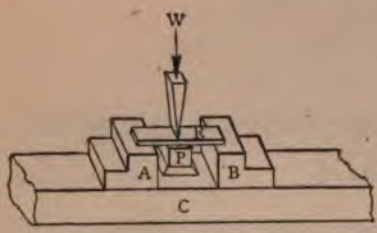

FIG. 297.

As the rod is bent an interference band coincides with the cross hair whenever the thickness of the air film is an odd multiple of $\frac{1}{2}\lambda$ for the light used, that is the depression corresponding to the interval between two successive bands coincident with a cross hair is for sodium light 589.5 millionths of a millimeter.

Denoting the central depression produced by any weight W by n the coefficient of extension in the direction of the length of the rod is given by the formula†

$$E = \frac{4bt^3}{Wl^3}n.$$

SURFACE OF EXTENSION COEFFICIENTS. Nine classes result from the measurements of extension coefficients of crystals.

1. *Isometric crystals* all yield an extension surface symmetrical to nine planes and thirteen axes. The four central sections parallel to the octahedral planes are circles. The cubic axes are directions of maximum or minimum extension and the octahedral axes are correspondingly minimum or maximum.

2. *Hexagonal.* Classes 19, 22, 23, 24, 25, 26, 27. The extension surface is a surface of rotation around $\dot{c}$. Each vertical central plane and the horizontal central plane are planes of symmetry.

* *Wied. Ann.*, XVIII., 325, 1883.
†Groth, *Phys. Kryst.*, III. ed., 203.

3. *Hexagonal.* Classes 18, 20, 21. The extension surface has one circular section horizontally through the center. In general the shape is that of a rhombohedron with rounded edges and angles symmetrical to three vertical planes and to a ternary vertical axis. Fig. 298 shows the section of the surface for calcite made by a principal section normal to a rhombic face, the dotted lines being the directions of greatest and least extension. Fig. 299 shows the section normal to this.

FIG. 298.

FIG. 299.

4. *Hexagonal.* Classes 16 and 17. Like No. 3 but without the three vertical planes of symmetry.

5. *Tetragonal.* Classes 11, 12, 14 and 15.

6. *Tetragonal.* Classes 9, 10 and 13.

7. *Orthorhombic.* The extension surface symmetrical to the three

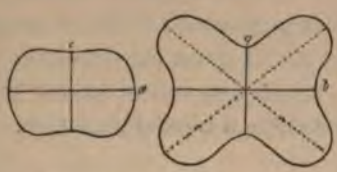

FIG. 300. FIG. 301.

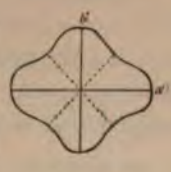

FIG. 302.

pinacoidal planes. Figs. 300, 301, 302, show these sections for barite.

8. *Monoclinic.* The extension surface symmetrical to the clinopinacoid and to the axis *b*.

9. *Triclinic.* No investigations are as yet recorded.

EFFECT OF PRESSURE IN ONE DIRECTION UPON THE OPTICAL CHARACTERS.

If a rectangular parallelopipedon* *C*, Fig. 303, is compressed between the parallel jaws of a screw press† in general the velocity of the light vlibrating parallel to the pressure is increased. If the pressure is uniform the convergent light effects are studied by bringing the specimen over the stage of a polariscope, but if at all un-

FIG. 303.

* Pockels used rectangular parallelopipeds 13 mm. high, 25.5 broad and thick and compressed from 30 k. g. with quartz to 2 k. g. for sylvite per sq. mm.

† For press recording pressure exerted see Groth, *Phys. Kryst.*, III. ed., 215.

equal, determinations in parallel light of vibration directions, faster and slower ray and strength of double refraction are more safe.

Amorphous Substances.

Glass* gives under compression a negatively uniaxial figure the optic axis parallel to the direction of pressure; that is c parallel $\mathfrak{a}$, or the ray vibrating parallel to the pressure is the more rapidly transmitted.

Optically Isotropic Crystals.

In general become biaxial but when the pressure is applied parallel to the cubic (quaternary) or octahedral (ternary) axes the crystal becomes uniaxial.

Optically Uniaxial Crystals (Denoting by $\mathfrak{a}, \mathfrak{b}, \mathfrak{c}$ the principal vibration directions and also velocities).

In positive crystals c is parallel to $\mathfrak{c}$ and $\mathfrak{a} = \mathfrak{b} > \mathfrak{c}$, while in negative crystals c is parallel to $\mathfrak{a}$ and $\mathfrak{a} > \mathfrak{b} = \mathfrak{c}$. Pressure parallel to c, in *positive* crystals, will cause $\mathfrak{c}$ to approach $\mathfrak{a}$ and possibly to equal (isotropy) or exceed it (negative); while in negative crystals pressure will only make $\mathfrak{a}$ still greater. In either positive or negative crystals pressure†, parallel a, will make the three velocities $\mathfrak{a}, \mathfrak{b}, \mathfrak{c}$ unequal, that is develope a biaxial structure.

Optically Biaxial Crystals.

The relations are more complex. Pressure perpendicular to the plane of the optic axis decreases the axial angle in positive crystals, and the crystal may be, as with heat, p. 170, for a certain pressure uniaxial or the axes may pass into a plane at right angles to their former position; with negative crystals the axial angle is increased. Pressure parallel to the obtuse bisectrix will in positive crystals increase the axial angle and in negative crystals decrease it.

CLEAVAGE.

In crystals the elastic limit varies with the direction, that is, in certain directions there is a weaker cohesion of the particles than there is in others, and many crystals tend to separate or *cleave*

* If the pressure is not uniformly applied at all points of the opposite surface the effect may resemble biaxial lemniscates.

† The centre of each ring system thus developed in quartz is colored, that is parallel to each optic axis there is a rotation, and it has been shown that in these directions two elliptically polarized waves are transmitted.

along more or less smooth plane surfaces normal to these directions of weaker cohesion. This is undoubtedly due to the fact that whatever form of regular grouping of particles may exist in a crystal, adjacent molecular planes in certain directions will be further apart than adjacent planes in other directions, and, therefore, held together with less force. For instance, in Fig. 304, it is evident that the closer the particles are along any direction *ab*, *cd*, *ef*, *gh* the greater is the distance to the adjacent row *a'b'*, *c'd'*, *e'f'*, *g'h'*, and as undoubtedly the further apart the particles are the weaker their cohesion, cleavage will be most likely to occur parallel to the planes in which the crystal molcules are most closely* packed.

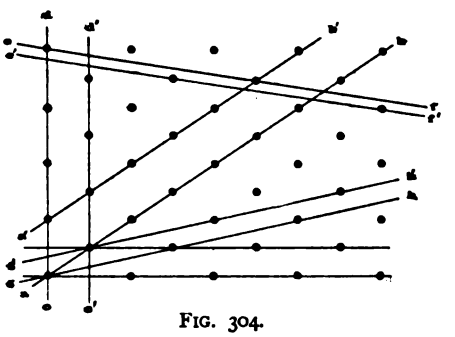

FIG. 304.

The same *general* relations exist for elastic and permanent deformation.† Cleavage will therefore be expected normal to the direction of greatest elastic extension, therefore parallel to faces with simple indices. The possible cleavage directions are:

System.	*Cleavage Form.*	*Examples.*
Isometric.	Hexahedron.	Galenite.
	Octahedron.	Fluorite, diamond.
	Rhombic Dodecahedron.	Sphalerite.
Hexagonal.	Basal Pinacoid.	Beryl, pyrosmalite.
	Hexagonal Prism.	Nephelite, apatite.
	Rhombohedron.	Calcite, siderite.
	Hexagonal Pyramid (rarely).	Pyromorphite.
Tetragonal.	Basal Pinacoid.	Apophyllite.
	Tetragonal Prism.	Rutile, wernerite.
	Tetragonal Pyramid (rarely).	Scheelite.
Orthorhombic.	Pinacoid.	Anhydrite, topaz.
	Prisms or Domes.	Barite.
	Pyramid.	Sulphur.
Monoclinic.	Clinopinacoid.	Orthoclase, gypsum.
	Basal pinacoid.	Muscovite, orthoclase.
	Orthopinacoid.	Epidote.
	Orthodome.	Epidote.
	Prism.	Pyroxene, amphibole.
	Pyramid (rarely).	Gypsum.

In triclinic crystals there can only be equally easy cleavage parallel to one plane. Nevertheless if there is a direction of nearly perfect cleavage one of the principal vibration directions will be approximately normal to this and the two others parallel thereto.

* The densest planes are usually the faces of common forms with simple indices.

† Groth, *Phys. Kryst.*, III. ed., 229.

GLIDING PLANES.

In those directions *ef*, *gh*, Fig. 304 in which the adjacent molecular planes are closest together and therefore most strongly held together it sometimes happens that under tangential pressure the particles glide or rotate, without separation, into a new position of equilibrium.

This phenomenon has been observed in calcite, pyroxene, anhydrite, stibnite and other minerals.

In calcite for instance let *ab'c'd* Fig. 305 represent a section through the optic axis normal to a rhombic face (see *abcd*, Fig. 222) then will *ac'*, the optic axis, make an angle of 63° 45′ with *ad*, the direction of $-\frac{1}{2}R$, and an angle of 45° 23½′ with *ab'* the short diagonal of a cleavage face.

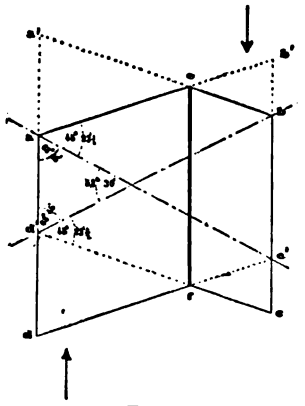

FIG. 305.

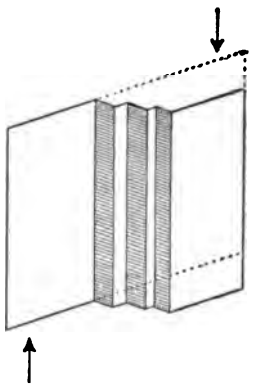

FIG. 306.

If pressure is applied gradually in the direction of the arrows, that is parallel to *ad*, there will be produced, about some plane *ef* parallel to *ad*, a gliding or rotation of the particles until *eb'* has taken the position *eb* at which $feb = 180° - feb' = 70° 51\frac{1}{2}'$.

In this rotated portion the optic axis will be *bd''* making with the optic axis *ac'* of the unchanged portion an angle of 52° 30′ as shown.

Instead of rotation about a single plane *ef* there may be rotation about several parallel planes as in Fig. 306 producing twin lamellæ.

If a little calcite cleavage is placed, as in Fig. 307, with an edge *ad* of a larger angle resting upon a steady support and the blade of a knife is pressed steadily down at some point *i* of the opposite

edge the portion of the crystal between t and c will be slowly pushed as indicated into a new position of equilibrium as if by rotation about $fghm$, or $-\frac{1}{2}R$, until the new face $gc'h$ and the old face gch make equal angles with $fghm$. If carefully done $gc'h$ will be a perfect plane, but more frequently it is step like and the rotated portion is apt to separate at the gliding plane.

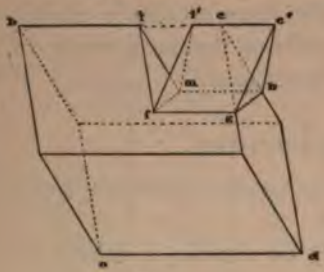

FIG. 307.

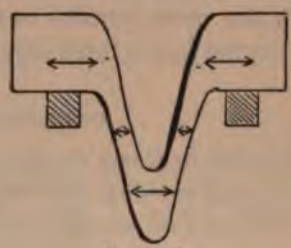

FIG. 308.

In the described phenomenon of gliding the particles evidently assume new relative positions. If however a rod of ice of square cross section is cut with one pair of long faces parallel to the optic axis the other perpendicular to the optic axis and a bending pressure, Fig. 308, applied normal to the axis there is no change in the direction of the optic axis, indicated by the arrows, nor any especial point at which the movement ceases; that is the particles have evidently slipped* without change in orientation. This has been called TRANSLATION.

PARTING.

The planes along which a slipping has occurred, although previously directions of maximum normal cohesion may thereafter be planes of easy separation or *Parting*,† differing from true cleavage, however, because in parting the easy separation is limited to the planes of actual molecular disturbance while true cleavage is obtained with equal ease in any part of the crystal.

FIG. 309.

PERCUSSION FIGURES.

If a rod with a slightly rounded point is pressed

* Mugge, *Neues Jahrb. f. Min.*, 1895, II., 211.

† Similar planes of easy separation may be due to other causes, for instance, during the growth of a crystal the planes at certain intervals may be coated with dust or fine lamellæ of a foreign substance and later the crystal may grow further. This may be repeated several times forming thus parallel planes of easy separation, *e. g.*, capped quartz.

against a firmly supported plate of mica and sharply tapped with a light hammer, three planes of easy separation will be developed as indicated by little cracks radiating * from the point, Fig. 309. One of these is always parallel to the ortho axis *b*, the others are at a definite angle *x* thereto of 53° to 56° in muscovite, 59° in lepidolite, 60° in biotite, 61° to 63° in phlogopite.

In halite, in the same way, on cube faces a cross is developed with the arms parallel to the diagonals of the face, while on an octahedral face a three-rayed star is developed.

CORROSION AND ETCHING.

Liquids or gases do not dissolve or attack chemically a crystal with equal rapidity in all directions.†

This may be shown experimentally by treating a sphere of the crystal in a solvent as, for instance, a sphere of quartz in hydrofluoric acid; or plates of different orientation may be placed for an equal time in the same solvent and their decrease in thickness compared.

Etch Figures.

The attack of any liquid or gas upon any crystal face does not commence at the same time at all points but proceeds from certain points first and later from others.

From each point of attack the action proceeds with different velocities in crystallographically different directions and if stopped at the right time the face will be found to be pitted with little angular etch cavities of definite shape. As these increase in number and size they finally reach a stage when they either touch or are separated only by little elevations, etch hills, the sides of which are the faces of the etch cavities.

Shape of Etch Figures.

Provided the same conditions exist of solvent, time and temperature, the etch figures developed on any one face of a crystal will be of the same shape and in parallel position plane for plane. On similar faces the etch figures will be alike, and on dissimilar faces will be unlike.

* By pressure alone three cracks diagonal to these are developed.

† Growth and solution appear to be reciprocal and the predominating faces of crystals growing in a solvent are the very planes which oppose greatest resistance to solution in that solvent.

The figures produced by solutions of different strength are not necessarily exactly the same, and with different solvents there may be a still greater difference,* as strikingly illustrated by the tests of Baumhauer † upon apatite with hydrochloric and sulphuric acids. Fig. 310 represents the etch figures in general, but

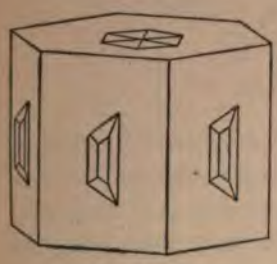

FIG. 310.

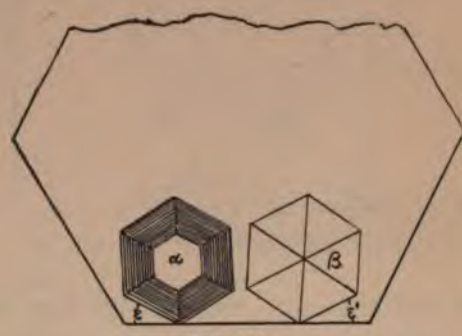

FIG. 311.

the basal planes treated with hydrochloric acid show side by side, dark deeply etched figures, usually consisting of a *negative* third order pyramid truncated by the base *a*, Fig. 311 ; and lighter less deep figures, usually a positive third order pyramid, *β*, Fig. 311.

With 100 per cent. acid the dark *a* figures are negative, but approximately second order pyramid, while the lighter images *β* are positive and also near the second order. If weaker solutions are used, the figures are differently oriented. With sulphuric acid still different orientations are obtained.

The following are the average values obtained for *ε*, Fig. 311, for different strengths, 100% HCl being Sp. Gr. 1.130 and 100% H_2SO_4 a Sp. Gr. of 1.836.

	100 per cent.	60 per cent.	50 per cent.	20 per cent.	10 per cent.	5 per cent.	1 per cent.
a figures HCl	(—) 27°20′	(—) 22°57′	(—) 20°48′	(—) 18°44′	(—) 18°21′	(—) 18°5′	(—) 17°34′
β figures HCl	(+) 27°10′	(+) 28°56′	(+) 26°40′	(+) 28°31′	(+) 28°31′	(—) 27°41′	(—) 17°34′
figures H_2SO_4	(+) 13° 7′	(+) 8°35′	(+) 11 0 5°	(—) 19°39′	(—) 16°52′	———	(—) 13° 6′

SYMMETRY OF ETCH FIGURES.

Whatever the solvent, the etch figures conform to the symmetry of the class to which the crystal belongs, and are rarely the limit

* For this reason it is not safe to assume any relation between the shapes of the etch figures and of the crystal molecule.

† *Ber. Ak. München*, 1887, p. 457.

forms common to several classes. Although the first developed etch figures may be the simple forms bounded by the planes of greatest resistance to solution, these are soon modified by other planes with complex indices, because the saturated solution in each cavity

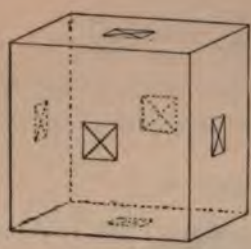

FIG. 312.

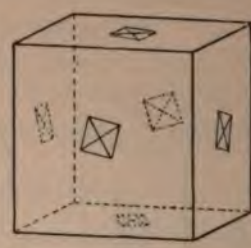

FIG. 313.

is replaced much more slowly near the bottom than near the border, and the attack is slower. The etch figures serve, therefore, as an important means (perhaps next to geometric form the *most* important) for determining the true grade of symmetry of a crystal. For instance, Figs. 312 and 313 show the shape and direction of the etch figures on cubes of galenite and sylvite, respectively. Fig. 314 shows the planes and axes proper to class 32, the highest grade of symmetry in the isometric system. Evidently the etch figures of galena satisfy all of these, whereas those of sylvite are not symmetrical to the planes of symmetry, but are to the axes. That is, galenite belongs to class 32, sylvite to class 29.

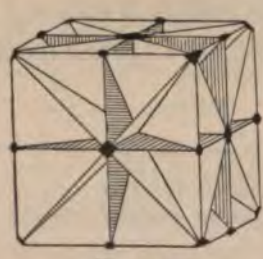

FIG. 314.

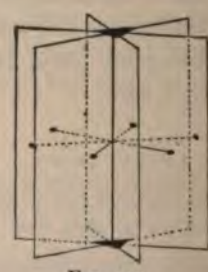

FIG. 315.

Similarly Figs. 316 and 317 show the shape and direction of etch figures on rhombohedra of calcite and dolomite, respectively. Fig. 315 shows the planes and axes of symmetry of class 21. The etch figures of calcite satisfy all of these, while those of dolomite are

not symmetrical to the planes of symmetry, but are to the axes. That is, calcite belongs to class 21, dolomite to class 17.

FIG. 316.

FIG 317.

Similarly, the etch figures of wulfenite Fig. 318 show the mineral to belong to class 10, and the etch figures of pyroxene Fig. 319 show it to belong to class 5.

MANIPULATION.

No general rule can be given, it being principally a question of ease of solution. The operation may consist merely in a slight pressure from a rag moistened with water, or the sliding of the crystal across a moistened spot in smooth filter paper, or, more frequently, the crystal is immersed for various periods in one of the mineral acids, caustic alkalis, or the mother liquor, hot, warm or cold, dilute or concentrated. High pressure steam, water, hydrofluoric acid, a pasty solution of caustic potash at 100° to 150° C., or even a red-hot fusion of acid potassium sulphate and fluorite have been used.

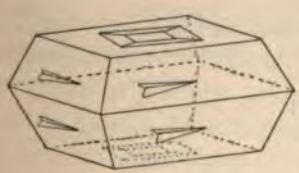

FIG 318.

FIG. 319.

Natural faces are usually most satisfactory for etching; cleavages are sometimes successfully etched. The etch figures are usually microscopic, and if large are not apt to be sharp and distinct.

CORROSION FACES.

The continued action of a solvent may dissolve away certain crystal edges, replacing them by planes conforming in symmetry to that of the crystal. These planes have been called corrosion faces.

HARDNESS.

The resistance of a smooth plane surface to abrasion is called the hardness, and is commonly recorded in terms of a decimal scale of ten common minerals selected by Mohs. In the light of the later more exact methods it is seen that there is no even approximately common difference in hardness between neighboring members* of this scale.

More exact methods, in which greater uniformity in pressure, cutting edge, condition and position of surface, etc., have been used by various experimenters. Exner† and others supported the crystal upon a little carriage moving on a horizontal track. The face to be tested was horizontal, the pressure p on the scratching point of steel or diamond was vertical and the carriage was moved back and forth by a weight W. The method preferred by Exner was with W constant to determine the pressure p necessary to produce a visible scratch.

From the results obtained Exner constructed the so-called hardness curves, for instance, Fig. 320 represents a dodecahedral cleavage face of sphalerite, radii are laid off from the center, each of a length proportionate to the value of p, for that direction and by connecting their ends a symmetrical figure is obtained, showing six directions of maximum and six of minimum hardness.

* Jaggar gives the following comparison of the hardness of the minerals of the Mohs scale *Amer. Journ. Sci.*, IV., 411, 1897:

Scale of Mohs.	Pfaff by boring with a diamond point.	Jaggar by boring with a diamond point.	Rosiwal by grinding with a standard powder.
1. *Talc*, laminated	———	———	———
2. *Gypsum*, crystallized	12.03	.04	.34
3. *Calcite*, transparent	15.3	.26	2.68
4. *Fluorite*, crystalline	37.3	.75	4.70
5. *Apatite*, transparent	53.5	1.23	6.20
6. *Orthoclase*, white cleavable	191.	25.	28.7
7. *Quartz*, transparent	254.	40.	149.
8. *Topaz*, transparent	459.	152.	138.
9. *Sapphire*, cleavable	1000.	1000.	1000.

† *Harte an Krystallflächen*, 38, 60, 103, 164.

Similarly in Fig. 321 a basal plane of barite shows a figure of lower symmetry.

The results of Exner show that the variations in hardness observed in any crystal are dependent upon the cleavages. Faces not cut by cleavage planes have constant hardness in all directions, while faces intersected by cleavage planes show minimum hardness parallel to the intersection with the cleavage plane and if the cleavages are of unequal ease the minimum hardness is parallel to the plane of easiest cleavage. So absolute is this relation that it may be expressed algebraically.

Very similar apparatus was used by Franz and Turner,* the mineral, however, being fixed and the point moving.

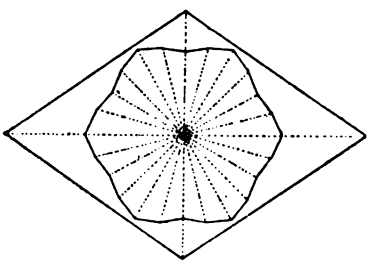

FIG. 320.

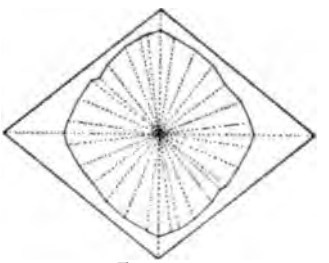

FIG. 321.

Pfaff† drew a diamond splinter of definite shape 100 times back and forth in one place, then shifted and repeated, widening the groove. The loss in weight of the crystal for the same number of movements of the diamond over the same area and with a constant pressure serve as approximate values for hardness, *i. e.*, hardness is inversely as the loss in weight. Pfaff also used a revolving diamond point. For equally deep penetration the hardness was as the number of revolutions.

Jaggar‡ designed an attachment to the microscope, in which the point of a cleavage tetrahedron of diamond is rotated at a uniform rate and under uniform pressure until a cut of uniform depth is obtained (measured by focusing on the rulings of a Zeiss micrometer glass, which is slightly inclined and follows the downward movement of the diamond point). The number of rotations of the point varies as the resistance of the mineral to abrasion by diamond.

* *Proc. Phil. Soc. Birmingham*, 1886.

† *Ber. Ak. München*, 1883, 1884.

‡ Microsclerometer, for determining Hardness of Minerals.—T. A. Jaggar, Jr.—*Amer. Jour. Sci.*, IV., Vol. IV., 399, 1897.

The effect of grinding has also been used as a test for hardness, for instance, Jannetaz and Goldberg * employed a so-called "Usometer," consisting of a rotating grinding disc, upon which four plates, the hardness of which is to be determined, were pressed by normally acting weights, the loss of weight of each giving the relative hardness.

THE METHODS OF STATIC PRESSURE.

Following the definition of hardness given by Hertz, Auerbach devised a method in which a plano convex lens of any mineral is pressed against a horizontal plate of the same mineral. Both are bent and touch throughout a circular space, and for some pressure P the elastic limit is reached, at which there is produced, if the mineral is brittle, a circular fissure, or, if the mineral is tough, a circular permanent indentation.

According to Auerbach † the limit pressure P_1 upon a square mm. of surface varies with the radius ρ of the lens, but the product of P_1 into the cube root of this radius ρ is essentially constant and may be called the Absolute Hardness, that is

$$H = P_1 \sqrt[3]{\rho}.$$

On this basis he determines ‡ the absolute values of the Moh's scale to be: corundum, 1,150; topaz, 525; quartz, 308; orthoclase, 253; apatite, 237; fluorite, 110; calcite, 92; gypsum, 14.

* *Ass. franc. p. l. avanc. d. sc.*, IX., Aug., 1895.

† F. Auerbach.—*Wied. Ann.*, XLIII., 61; XLV., 262, 277.

‡ *Wied. Ann.*, LVIII., 357.

APPENDIX.

Suggested Outline of a Course in Physical Crystallogrphy.

PRELIMINARY EXPERIMENTS.

In order to secure systematic work the following outline of experiments has been prepared:

A. Geometrical Characters.

Preliminary practice should be given with crystal models in the study of the thirty-two classes and the Miller indices, also in the use of the zonal equations and in stereographic projections. The hand goniometer and the 743 model set of Krantz furnish excellent practice.

1. Measurement of Crystals. (Pp. 62–75.)

Practice should first be given in accurate adjustment and measurement of a known angle, for instance the cleavage angle of calcite.

Crystals of known system may then be measured, the elementary planes (p. 13) being chosen and, by a preliminary examination, the zones noted and the angles selected which it will be necessary to measure in order to determine the elements of the crystals and the indices of the faces.

Practice should, if possible, be given:

(*a*) With horizontal circle goniometer. Babinet or Fuess.

(*b*) With vertical circle goniometer. Wollaston, Mallard or Groth.

*(*c*) With two circle (theodolite) goniometer.

2. Determination of Elements, in part by spherical triangles (p. 6), in part by special formulas. (Pp. 13–15, 34, 38, 45, 56.)

3. Determination of Indices and Angles. In so far as possible by zonal equations (pp. 17–20) but also by spherical triangles and special equations.

4. Crystals Projection or Drawing.

In addition to the free-hand perspective and stereographic projections needed in the measuremeut, accurate drawings by the various methods should be prepared, especially:

(*a*) Stereographic projection. (Pp. 20–24.)

(*b*) Clinographic prospective. (Pp. 76–84.)

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* Chas. Palache. *Am. Jour. Sci.* S. 4, Vol. II., p. 279, 1896.

B. Optical Characters.

5. Double Refraction and Polarization. (Pp. 97-100.)

With mounted calcite rhombs. (P. 97.)

(*a*) A ray of common light through one rhomb, is split into two rays of approximately equal intensity which remain always in a principal section.

(*b*) If one of these rays is sent through a second calcite rhomb, the two resultant rays vary in intensity as if due to a resolution of a vibration parallel (or perpendicular) to the principal section of the first rhomb, into components parallel and perpendicular to the principal section of the second rhomb.

6. Determination of Indices of Refraction, with monochromatic light.

Graphic determination (pp. 86–90) of the direction of the refracted ray and of the ray incident at the critical angle should be made with assumed indices, after which measurements should be made both upon singly and doubly refracting crystals by:

(*a*) Prism method. (Pp. 88–90, 103, 146–147.)

(*b*) Total reflection methods. (Pp. 90–95, 104, 147d.)

(*c*) Displacement method. (Pp. 120.)

If singly refracting, the substance will yield the same index for all directions. If doubly refracting the principal indices will only be yielded in certain directions.

7. Production of Polarized Light. (Pp. 105–110.)

Examination of Nicol and Hartnack prisms (double refraction and total reflection), tourmaline pincers (double refraction and absorption), glass plate polarizers (reflection or refraction). Polariscopes (combinations of two polarizers), including older types Norremberg, and later types of Universal Apparatus, and Polarizing Microscopes.

8. Determination of Extinction (Vibration) Directions. (Pp. 117, 145.)

With polariscope (or polarizing microscope) and monochromatic parallel light, sections between crossed nicols are black when, Fig. 237, $\varphi = 90°$, 180°, 270°, 360°, and midway between show brightest illumination.

With white light use a test plate (gypsum red) or special eye piece. Results are less accurate if there is marked dispersion.

The section, if too thick, will admit rays at directions not normal, preventing perfect extinction.

Sections of enstatite and pyroxene illustrate respectively parallel and oblique extinction.

9. INTERFERENCE OF PLANE POLARIZED RAYS. (Pp. 110–115.)

(*a*) With monochromatic parallel light and crossed nicols, extinction (throughout revolution) takes place when $\Delta = \lambda$, 2λ, 3λ, etc., and for $\Delta = \frac{1}{2}\lambda$, $\frac{3}{2}\lambda$, $\frac{5}{2}\lambda$ there is the brightest illumination.

(*b*) With white parallel light Newton's colors of 1°, 2°, 3°, etc., order result.

Wedges of quartz, gypsum, or mica, or plates of different thickness of any mineral serve for illustration.

10. DETERMINATION OF Δ AND $n_1 - n$. (Pp. 118–119.)

By the v. Federow mica wedge, quartz wedge or Babinet compensator Δ is found and checked by color chart. Obtain $n_1 - n$ by formula $\Delta = t(n_1 - n)$, t being measured.

Use the sections of tests 8 and 9.

11. DETERMINATION OF VIBRATION DIRECTION OF FASTER RAY. (Pp. 118.)

By test plate of mica or gypsum or by wedge of quartz or mica.

Use the sections of tests 8 and 9.

12. OPTICALLY ISOTROPIC CRYSTALS.

Between crossed nicols, with either parallel or convergent light all sections remain dark throughout rotation. The index of refraction is independent of the direction.

Use sections garnet, hauynite, nosite, etc.

13. OPTICALLY UNIAXIAL CRYSTALS.

Models of positive and negative ray surfaces and indicatrices should be studied. (Pp. 100–102.)

(*a*) *Sections normal c between crossed nicols.* In parallel light the section (if not too thick) remains dark throughout rotation. In convergent monochromatic light the arms of the cross remain fixed during rotation of the plate, the rings are wider apart in thinner sections, and with white light are colored in the order of Newton's colors. (Pp. 110–117.)

(*b*) *Sections parallel c:* interference hyperbolæ in monochromatic light and approximate indices by displacement. (Pp. 116.)

(*c*) Sections oblique to *c*. (Pp. 110, 112, 116.)

(*d*) Determine character of ray surface by different methods. (Pp. 120–121.)

Sections of zircon, quartz, calcite, beryl, and apatite form a good series.

14. UNIAXIAL CYSTALS IN WHICH *c* IS A DIRECTION OF CIRCULAR POLARIZATION. (Pp. 122–130).

(*a*) Circular and elliptical polarization, with Fresnel rhombs.

(*b*) Sections normal *c*, with parallel light.

Monochromatic, determine the direction and amount of rotation of analyzer to produce extinction.

White, note the sequence of colors on rotation of the analyzer.

(*c*) In section normal *c*, with convergent light. (Pp. 127.)

Monochromatic, interference figure, giving the inter-wound spirals with ¼ undulation plate, or with superposed right and left sections.

White light, interference figures with colored center, changing on rotation of analyzer in the same sequence as for parallel light.

(*d*) Sections parallel or oblique to *c* essentially conform to 13 (*b*) and (*c*).

Use sections of quartz and cinnabar. Compare thickness of quartz wedges cut respectively parallel and normal to *c*, which give the same color.

15. BIAXIAL CRYSTALS.

Models of positive and negative ray surfaces and indicatrices should be studied. Given three principal indices of refraction for yellow light construct the three optical principal sections. (Pp. 132–137.)

(*a*) In sections normal to the acute bisectrix.

1. *In parallel light* essentially as in tests 8 and 9. The relation of the vibration (extinction) direction to any cleavage or crystalline outline should be noted. (Pp. 137, 138.)

2. *In convergent monochromatic light* leminiscates depend upon thickness, but the distance apart of the axial points does not. (Pp. 138, 139.)

3. *In convergent white light* determine character of dispersion. (Pp. 140–144.) Examine models of dispersion.

4. Determine character of Ray Surface. (Pp. 154.)

Sections of aragonite, niter, cerussite, barite, calamine, topaz, gypsum, titanite, diopside, orthoclase, borax, heulandite, muscovite and phlogopite illustrate all phases.

(*b*) In sections normal to an optic axis in parallel monochromatic light there is a uniform illumination of the field, in convergent monochromatic light will be seen rings and one dark arm, which rotates in opposite direction to the stage. (Pp. 137–139.)

*16. ORIENTATION OF a, b and c, IN BIAXIAL CRYSTALS. (Pp. 145, 146.)

By extinction directions and interference figures in variously oriented pairs of parallel planes (natural faces, cleavages or planes obtained by grinding).

Use suite of sections or crystals of one substance.

17. DETERMINATION OF ANGLE BETWEEN OPTIC AXES. (Pp. 148–154.)

(*a*) By measurement of apparent angle in sections normal to the bisectrices either by rotation about b, with universal apparatus or its equivalent, in air or in a liquid of known index (pp. 148–150, 152), or by measurement of the distance apart of the emerging axes in a microscope or polariscope. (Pp. 151.)

(*b*) By measurement with entire crystal in a liquid of approximately or exactly the same index as the crystal. (Pp. 151.)

Use sections of muscovite and complete crystals.

18. ABSORPTION AND PLEOCHROISM. (Pp. 96, 130, 131, 155–159.)

In isotropic crystals there is no pleochroism, but there may be either color (unequal absorption) or no color (equal absorption). The absorption is independent of the direction.

In uniaxial crystals there is no pleochroism for transmission parallel *c*, but there may be pleochroism in other directions with greatest differences for transmission normal to *c*.

In biaxial crystals the greatest differences in color are for rays vibrating parallel to certain directions which in orthorhombic crystals are *a b c*, in monoclinic crystals, one is *b* and in triclinic they are not related to the crystal axes.

Sections of vesuvianite, tourmaline, penninite, iolite, andalusite, chrysoberyl, epidote, titanite and axinite.

Absorption tufts in epidote and andalusite.

*19. MICA COMBINATIONS. (Pp. 161–162.)

Production of uniaxial interference figure and rotation of plane of vibration by piling mica plates.

C. THERMAL CHARACTERS.

20. CONDUCTIVITY. (Pp. 164–165.)

Illustrate surface conductivity by methods of Röntgen and Voigt, obtaining circular figures in isotropic or in uniaxial normal to *c*, but in other sections ellipses in which note relation of axes of ellipse to crystal axes.

*21. EXPANSION. (Pp. 166–171.)

Indirect determination by observing the effect of heat upon the optical characters, especially the effect upon the position of the optic axes. (P. 170.)

Sections of gypsum.

Indirect determination by observing change in diedral angle with calcite cleavage. (P. 169.)

D. ELECTRICAL AND MAGNETIC CHARACTERS.

22. MAGNETIC INDUCTION. (Pp. 172, 175.)

Suspension between the poles of an electromagnet of a small mass no too unequal in dimensions.

Isometric Crystals indifferent equilibrium for all positions.

Tertragonal or Hexagonal Crystals.

With c vertical all positions are in indifferent equilibrium, but with c horizontal it takes either axial or equatorial position.

Note + and — paramagnetic or diamagnetic character.

Crystals without any axis of magnetic isotropy (orthorhombic, monoclinic, triclinic).

Determine position and relative strengths of magnetization axes. (P. 173.)

Experiments illustrating electrical transmission (p. 175), conductivity (p. 175), thermoelectric currents (p. 176), or dielectric induction (p. 177), may be devised when apparatus is available.

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23. PYROELECTRICITY. (Pp. 180–182.)

Qualitatively by Kundt's dusting with sulphur and minium upon crystal while undergoing uniform change of temperature.

Quantitatively by self-charging electrometer.

Crystals of tourmaline and quartz.

24. PIEZOELECTRICITY. (Pp. 182–183.)

Qualitatively by Kundt's method, developing the strain either by unequal heating, or by direct pressure.

Basal Sections of Quartz.

E. CHARACTERS DEPENDENT UPON ELECTICITY AND COHESION.

27. ELASTIC DEFORMATION FROM PRESSURE IN ONE DIRECTION. (Pp. 185–188.)

Effect of pressure in one direction upon the optical characters of cubes of glass and of different minerals.

28. CLEAVAGE ANG GLIDING PLANES. (Pp. 189–191.)

In Calcite, Pyroxene, Stibnite and the Micas.

29. ETCH FIGURES (192–196.)

Examination of suite of *Apatite, Calcite, Calamine,* etc.

SYSTEMATIC EXAMINATION* OF THE CRYSTALS OF ANY SUBSTANCE.

The objects are to obtain a record of the physical characters for different directions, and to determine, from the physically equivalent directions, the grade of structural symmetry.

The best crystals (p. 69), are chosen, the symmetry judged by examination with a hand glass or microscope and checked by an approximate

* Based upon Groth, *Phys. Kryst.*, III., 537–543.

determination of extinction directions (p. 117, 145), and possibly of interference figures (115, 127, 138). Great assistance may here be given by a rotation apparatus (p. 151, 152).

The crystals are sketched and letters assigned to the faces. Elementary planes (p. 13) are chosen parallel to cleavages (p. 189) or according to prominence.

Geometric Characters or Symmetry of Growth.

The crystals are mounted, centered and measured (pp. 63–70), by zones, precedence being given to the angles between the elementary planes. A record is kept, as described (p. 71), and also upon a free hand stereographic projection (p. 20). A quality mark is assigned (p. 70), to each reflection and used in the averaging.

The elements are calculated (pp. 30, 34, 38, 45, 56, 61) by the formulæ given or by spherical triangles (p. 6). The indices are determined in general by intersecting zones (p. 17–19), and all angles are calculated from the indices and elementary angles by solution of the spherical triangles (p. 6), in the stereographic projection, or by special formulæ, or by zonal equations, and compared with the measured angles.

One or more perspective drawings are then made. (Pp. 76–84.)

Etch Figures, or the Symmetry of Solution.

Experience shows that the symmetry of the etch figure is almost invariably the true structural symmetry of the crystal. More than one solvent should be tried and the conditions varied.

Optical Characters and Symmetry.

For transparent crystals the optical characters are the safest proof of symmetry, often proving apparently simple crystals to be complex.

The optical constants for transmission are α, β and γ (133) for light of different wave-length (89) and their orientation (145) and direct measurement by the prism method (88, 103, 146) or by total reflection (90, 104, 147) for light of each of Frauenhofer's lines is sometimes made.

More frequently the determination is limited to the orientation and to constants dependent on α, β and γ, such as retardation (118), strength of double refraction (119), character of ray surface (120, 154), and, if biaxial, angle between optic axes (148). These determinations are for sodium flame and possibly for lithium and thallium (89). If biaxial the interference figure should also be observed in white light for dispersion (132–144).

Absorption and pleochroism are determined (96, 130, 155).

If the crystals show a rotation of the plane of polarization (123) the amount (129) and direction (128) is determined.

THE REMAINING PHYSICAL CHARACTERS.

The most frequently used tests remaining are the effects of heat, electricity and pressure upon the optical characters. In addition, however, and especially in minerals opaque to light the following may be determined without very elaborate apparatus.

(*a*) The heat conductivity, Röntgen or Voigt methods (p. 164).

(*b*) The pyroelectric charges, Kundt's method (p. 180).

(*c*) Cleavage (p. 189), gliding planes (p. 190), percussion figures (p. 192).

Thermal expansion, Magnetic Induction, Electrical conductivity, Thermoelectric currents, Dielectric induction, Elastic deformation rarely form part of an investigation.

The ascertained facts may be conveniently recorded as suggested by Groth* and constitute a Crystallographic description.

(*a*) Class of Symmetry.

(*b*) Geometric Characters.

Elements, Enumeration of forms and description of habit with perspective drawings, Twinnings, Conditions of temperature solution, etc., in production of crystals.

(*c*) Cleavages, Tabulation of measured and calculated angles, Gliding Planes and Etch Figures.

(*d*) Optical Characters.

(*e*) Other physical characters.

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* *Phys. Kryst*, III., 543.

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